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A Bi-Level Model for Optimal Placement and Sizing of EV Fast Charging Stations Considering Traffic and Power Network Interactions

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***Abstract*— This paper presents a bi-level optimization model for the siting and sizing of electric vehicle fast charging stations (FCSs), considering the constraints of the power distribution network. In the presented method, queuing theory and a user equilibrium-based traffic assignment model are used to determine the size of FCSs. The upper-level problem aims to maximize the profit of the FCS owner by determining optimal locations and capacities of FCSs. The lower-level problem minimizes the operational cost of the distribution network while considering power flow constraints and EV charging demands. The bi-level model is transformed into a single-level mathematical program using the Karush-Kuhn-Tucker (KKT) primal-dual optimality conditions of the lower-level problem due to the linearity of the LL problem. Simulation results on the IEEE 33-bus distribution system and a 25-node transportation network show that two FCSs are optimally installed at buses 25 and 32 with 9 and 7 chargers, respectively, yielding a daily profit of approximately $6,147 for the investor. Sensitivity analysis demonstrates that higher electricity selling prices lead to increased profitability and expansion of charging infrastructure, highlighting the effectiveness of the proposed framework in capturing the economic interaction between the DSO and private investors.**

Keywords: Bi-level Optimization, Karush-Kuhn-Tucker, Queuing Theory, Sizing of Electric Vehicle Charging Stations, Locational Marginal Prices

NOMENCLATURE

Sets and indices

r	Travel origin index
u	Travel destination index
k	The path index for moving from origin r to destination u

Parameters

fr_a	Traffic load of the road a
t_{a0}	Travel time on the road without traffic
a_K^F	Fixed cost in bus k
a_K^{LS}	Cost dependent on location and size of charging station on bus k
a^{CHF}	Cost of equipment required to build an FCS
C	Overall daily charging demand (times/day).
σ	percentage of EVs that are recharged at home by using standard wall outlets (%)
β	Choosing the ratio of charging posts

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f_t^{trip} Trip ratio in time t

$f_{k,t}$ Traffic flow captured by the kth FCS

μ Mean service rate of FCS (vehicles/hour)

q_{ru} Total traffic load between source r and destination u

ε Interest rate

n_{FCS} capital recovery factor

p^{FCS} Nominal charging power of the fast charging facility.

variables

$fp_{r,u,k}$ Traffic load of the kth path between origin r and destination u

t_a Travel time on the road a (hour)

$Fn_{k,t}$ The total traffic load of the arcs ending at node k

$\delta_{r,u,k,a}$ Binary variable indicating the existence of arc a on the kth path for moving from origin r to destination u

$\rho_{k,t}$ Occupation rate of fast charging facilities of the kth FCS in time t (%)

ρ_k^{RH} Occupation rate of fast charging facilities of the kth FCS in the rush hour (%)

Z_k Number of charges at the kth station

λ_k^{RH} The number of vehicles entering the candidate charging station during peak hours (vehicles)

W_k^{RH} Average waiting time for the charging service in the th FCS during the rush hour (hour)

$P_{k,t}^{FCS}$ Charging power of the kth FCS in time t (kW)

z_k Size of the kth FCS

R^{FCS} The revenue of the FCS investor ($)

$Cost^{FCS}$ The cost of the FCS investor ($)

C^{inv} The investment cost of FCS ($)

U_i Binary variable representing the establishment of an FCS at bus i

$Cost_{i,t}^{up}$ The cost of buying energy from the upstream network for the FCS investor ($)

$P_{g,t}$ Active power generation of generator g in time t (kW)

$P_{i,t}^{grid}$ Active power purchased from the electrical grid at bus i, in time t (kW)

$P_{i,t}^{LS}$ The amount of load shedding at bus i (kW)

$P_{l,t}^{flow}$ Active power flow through line l at time t (kW)

$\theta_{i,t}$ Voltage angle at bus i, in time t

Acronyms

DSO distribution system operator

EV Electric Vehicles

FCS Fast-charging stations

KKT Karush-Kuhn-Tucker

LL Lower level

OPF Optimal Power Flow

OD Origin Destination

PSO Particle Swarm Optimization

UL Upper Level

I. INTRODUCTION:

The growing adoption of Electric Vehicles (EVs) highlights the urgent need to expand fast-charging infrastructure. Fast-charging stations (FCS) significantly reduce charging time, enhance travel convenience, and promote EV acceptance. However, their deployment requires careful planning due to high investment costs and grid capacity limits. Strategic integration of fast-charging stations within transportation and power networks ensures efficient energy delivery and supports the transition toward sustainable, low-emission transportation.

Recent studies have proposed various optimization approaches to address the complex problem of locating and sizing FCSs for electric vehicles. A bi-level optimization model is proposed in [1] to determine optimal fast-charging station locations in a metropolitan network, minimizing travel time and infrastructure costs while considering vehicle types and traffic congestion. Reference [2] uses a genetic algorithm to identify profit-maximizing locations and designs for fast EV charging stations, considering stochastic charging demand, user-equilibrium traffic, and the interdependence between congestion, station queues, and price-sensitive charging behavior. Reference [3] develops an optimization strategy for allocating FCSs for electric vehicles. The proposed mixed-integer programming model minimizes investment and operating costs while considering PV-integrated carports and battery energy storage systems as alternative planning options.

When the distribution system operator (DSO) owns and operates the FCSs, the planning problem becomes integrated, combining investment and operational decisions under network constraints. Reference [4] presents a multi-objective model to determine the optimal placement and sizing of FCSs along intra-city corridors, integrating transportation and electrical networks. Reference [5] addresses the optimization of FCS size and location, taking into account investment, operation, and maintenance costs, power system losses, and reliability costs. A Particle Swarm Optimization approach is employed to identify the optimal station sizes and locations. In [6], a strategic framework for ultra-fast EV charging station planning is proposed, optimizing locations and charger numbers using the Voltage Stability Index and Harris Hawk Optimization to minimize total costs. The model also accounts for uncertainties in charging behavior and electricity prices via the 2m-Point Estimate Method. In [7], an optimal allocation and sizing method for EV charging stations in the Allahabad distribution network is proposed. The approach minimizes installation costs while enhancing grid performance based on voltage profile and real and reactive power loss indices. The nonlinear mixed-integer problem is solved using an improved metaheuristic algorithm, the Balanced Mayfly Algorithm. In [8], a multi-objective optimization approach is presented for the optimal placement of FCSs, DGs, and shunt capacitors. A Pareto-based hybrid method combining Grey Wolf Optimizer and Particle Swarm Optimization is employed to minimize multiple objectives in a 118-bus radial distribution system.

Several studies have formulated the siting and sizing of FCSs as bi-level optimization problems to capture the interaction between DSO, FCS investors, and EV users. In [9], a bi-level multi-objective model is developed for EV charging station location planning, simultaneously considering user preferences and waiting times. The upper level optimizes station locations and capacities to minimize total cost and service delay, while the lower level allocates users to stations to minimize travel time. In [10], a bi-level optimization model for fast charging station allocation is proposed. The upper layer maximizes investor profits, while the lower layer coordinates the expected efficiency of the charging service supply. In [11], an online vehicle-charging assignment model is integrated into the fast-charging station location problem for dynamic ridesharing with electric vehicles. The bi-level optimization aims to minimize the fleet's total daily charging time. In [12], a bi-level optimization model is developed for the location and sizing of EV charging stations by jointly considering transportation and energy demands. The lower level incorporates user equilibrium traffic conditions as constraints, while the upper level optimizes the location, capacity, and pricing of new stations alongside existing ones. In [13], a bi-level programming model is proposed to determine the optimal locations of EV charging stations, aiming to minimize drivers' range anxiety. In [14], a strategic charging-behavior-aware model is formulated as a bi-level mixed-integer program. The lower level models drivers' charging responses using a network equilibrium approach, while the upper level optimizes charging station location and sizing to minimize overall traffic time and investment costs. In [15], a bi-level optimization model addresses the strategic location and sizing of EV charging stations under stochastic vehicle flows and charging times. The upper level minimizes infrastructure costs while ensuring probabilistic service requirements on users' waiting times, considering route choice responses. In [16], a bi-level planning model for EV charging stations is proposed, incorporating traffic conditions and energy consumption per unit distance. The lower level represents users' charging decisions, while the upper level optimizes station location and capacity. The model is solved using the Improved Whale Optimization Algorithm and Voronoi diagrams. In [17], a bi-level optimization model considers the impact of non-system-optimal driver behavior on EV charging station capacity. The upper level addresses the provider's station location decisions, while the lower level models drivers' selfish charging choices to minimize stops. Reference [18] proposes a bi-level model where the lower level minimizes daily operating costs through bus scheduling and charging optimization, while the upper level designs charging stations using a tabu search algorithm

Several other studies have focused on bi-level optimization models that primarily consider the transportation network perspective, emphasizing traffic flow, user behavior, and route planning in EV charging station deployment. In [19], a bi-level optimization approach using Particle Swarm Optimization is proposed to determine optimal EV charging station locations while minimizing losses and operating costs. An integrated EV charging planning algorithm manages connections to avoid peak load issues and severe voltage drops. In [20], a MILP-based coordinated planning method is proposed for coupled power and transportation networks, optimizing new road deployment, EV charging station placement along these roads, and power network expansion to support the stations. In [21], a bi-level planning model considers both investor costs and user satisfaction. The upper level minimizes construction costs and network losses using an improved Particle Swarm Optimization, while the lower level evaluates user satisfaction by minimizing travel time and expenses, considering queue times and distances through Dijkstra's algorithm and queuing theory. In [22], a bi-level programming model determines optimal EV charging station allocation in the presence of wind turbines. The upper level maximizes station profit, while the lower level minimizes power losses using available sources and dynamic feeder reconfiguration. The impacts of cryptocurrency miners and demand-side management are also considered. In [23], a combined road transport and electric distribution network model is proposed for strategic EV charging station deployment. A bi-level optimization approach minimizes user travel costs, power losses, and voltage deviations, employing PSO-DS for station placement, convex optimization for traffic equilibrium, and AC OPF for grid operation. In [24], a bi-level EV charging station planning model considers spatiotemporal load distribution under uncertainty. The lower level predicts charging demand using OD matrices, dynamic Dijkstra routing, and LHS, while the upper level minimizes station planning costs and user behavior, and also accounts for distribution operation costs and emissions from uncertain renewables. In [25], a bi-level planning model for EV charging stations in coupled distribution-transportation networks is proposed to enhance post-fault security. The upper level optimizes station locations and capacities, while the lower level designs EV charging routes to minimize overall travel costs.

Investment in FCSs is typically made by private investors whose economic objectives often differ from those of the DSO. Although many studies have addressed the siting and sizing of FCS, most focus on either transportation behavior or distribution network operation, and do not capture the economic interaction between private investors and the DSO, especially when electricity prices are determined through network-constrained optimal power flow. To address this gap, this paper proposes a multi-objective bi-level model that jointly determines FCS locations, capacities, and energy exchange prices. The main novelties of this study are as follows:

TABLE I
Comparative Analysis of Different Articles with the Proposed Model

Ref number	Objective function	Decision Variables			Traffic model	Optim model		Stakeholders Considered		Problem Focus		Methode of optimization
		Location	Size	Price		Bi-level	single-level	FCS owner	DSO	electrical	Transport	
[1]	minimizing travel time and infrastructure costs	✓	-	-	re-routing behaviours of travellers	✓	-	✓	-	-	✓	cross-entropy method
[2]	Maximizing profit	✓	-	-	user-equilibrium traffic	-	✓	✓	-	-	✓	GA
[4]	maximizing the traffic flow coverage	✓	✓	-	Driving Range-Based Traffic Flow Capturing Model	-	✓	-	✓	✓	✓	improved PSO
[5]	Minimizing total costs	✓	✓		simple	-	✓	-	✓	✓	✓	PSO
[6]	minimize total costs	✓	✓		simple	-	✓	-	✓	✓	✓	Harris Hawk Optimization
[9]	minimize total costs	✓	✓	-	simple	✓	-	-	✓	✓	✓	
[10]	maximizes investor profits	✓	✓	-	`simple	✓	-	✓	-	✓	✓	KKT trans
[11]	minimize the fleet's total charging time	✓	✓	-	online vehicle-charging assignment	✓	-	✓	-	-	✓	A surrogate-assisted optimi approach
[12]	minimizes infrastructure costs	✓	✓	-	User Equilibrium traffic assignment	✓	-	✓	-	✓	-	-
[14]	minimize overall traffic time and investment costs	✓	✓	-	User Equilibrium traffic assignment	✓	-	✓	-	-	✓	Descent algorithm
[18]	Minimizing installation costs	-	✓	-	simple	✓	-	✓	-	-	✓	tailored column generation-based heuristic algorithm
[19]	minimizing losses and operating costs	✓	-	-	-	✓	-	-	✓	✓	-	PSO
[23]	minimizes user travel costs, power losses,	✓	✓	-	traffic equilibrium	✓	-	-	✓	✓	-	PSO
Prop model	**maximizes investor profits**	✓	✓	✓	**user-equilibrium traffic +Queing theory**	✓	-	✓	✓	✓	✓	**KKT trans**

1- The integration of FCS private investor and DSO objectives within a bi-level framework.

2- the simultaneous optimization of FCS location, sizing, and electricity exchange pricing under network constraints, with the bi-level model transformed into a single-level formulation using Karush-Kuhn-Tucker (KKT) conditions.

The rest of this paper is organized as follows: the bi-level model for determining the location and capacity of FCSs and the DC optimal power flow of the distribution network is presented in Section II. Simulation results and sensitivity analysis are presented in Section III, and Section IV concludes the paper results and future works.

II. BI-LEVEL OPTIMIZATION APPROACH

In this paper, a two-level model is proposed to represent the interaction between the charging station owner and the distribution network operator. Before implementing the two-level model, the charging demand of each candidate station is first determined by considering the user equilibrium-based traffic assignment model and queuing theory. In the two-level model and at the high level, by determining the energy purchase price from the distribution network at the location of each FCS and maximizing the profit of the charging station owner, the locations of the charging stations are determined. By determining the locations of the charging stations and adding the electric power demand of these stations to the distribution network, and to minimize the cost of energy production for the distribution network operator, the energy sales price at each busbar is determined. This price will be the same as the energy sales price to the charging stations installed on the same busbar. The energy sales price to the charging stations is transferred to the high-level problem as a known parameter, and this process will continue until the final answer is reached. This model is shown in Fig. 1. The user equilibrium-based traffic assignment model, the queue theory, and the UL and LL problems are formulated below.

A. The User Equilibrium-based Traffic Assignment Model

In the proposed method, traffic information is used to simulate the behavior of electric vehicles and estimate charging demand. However, raw traffic flow data cannot be directly used in the planning of charging stations. As a result, daily origin-destination data are used to generate traffic flows. To obtain these data, an optimal system allocation model is used to generate and allocate traffic flows on each route of the transportation network [26]. The objective of the optimal system allocation model is to achieve the minimum travel cost, according to equations (1a) to (1b).

$$\min \sum_{a} fr_a . t_a \tag{1a}$$

$$\sum_{k} fp_{r,u,k} = q_{r,u} \quad \forall r, \forall u \tag{1b}$$

$$fp_{r,u,k} \geq 0 \quad \forall r, \forall u, \forall k \tag{1c}$$

$$fr_a = \sum_{r}\sum_{u}\sum_{k} fp_{r,u,k} . \delta_{r,u,k,a} \qquad \forall r, \forall u, \forall k \tag{1d}$$

$$t_a = t_a^0 \left[1 + b \left(\frac{fr_a}{c_a} \right)^v \right] \tag{1e}$$

Equation (1a) is the objective function of the problem, which represents the minimization of the travel cost. Equation (1b) guarantees the principle of network flow conservation. This relation means that the sum of the flow of all paths between each origin-destination is equal to the travel demand of that origin and destination. In this relation, the condition of non-negativity of the traffic flow on the kth path between origin r and destination u is also considered. Equation (1d) indicates that the traffic flow on road *a* is equal to the sum of the flows on all paths that include road *a*. The travel time on road *a*, given the accumulated flow on this road, is shown in (1e).

B. Capacity of Candidate FCSs Based on Queueing Theory

Queuing theory is often used to mathematically analyze the outcome of random arrivals of customers to receive service from the system. After obtaining the equilibrium traffic flow of each route, the random movement of vehicles and the capacity of charging stations are analyzed using queuing theory. The charging station service system is considered an M/M/S queue system. Queue service models are represented by the abbreviation (A/B/C), where A represents the distribution between two consecutive arrivals. Since the arrival time of each vehicle at the charging station is a random variable, in this paper, the arrivals of these vehicles are considered as a Poisson distribution. The second term, B, represents the distribution of the service duration, which is assumed to follow a uniform distribution, and the third term

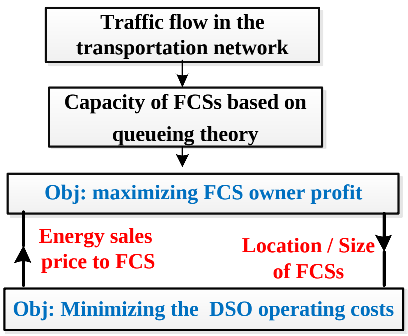

Fig. 1: The prposed bi-level model

represents the number of service providers (chargers at a station).

One of the important parameters in the Poisson process is its mean value. It is assumed that electric vehicles have a similar driving pattern to conventional vehicles and that the average arrival rate of vehicles at each FCS is proportional to the traffic flow attracted by that FCS. Thus, the mean arrival rate of EVs in the *k*th FCS at time t can be calculated as:

$$\lambda_{k,t} = C(1-\sigma)(1-\beta)\frac{f_t^{trip}}{\sum_{t} f_t^{trip}} \frac{f_{k,t}}{\sum_{k} f_{k,t}} \quad \forall k, \forall t \tag{2a}$$

The capacity of charging stations can be calculated as a nonlinear integer programming model, assuming that qμ is independent of the vehicle arrival rate, based on a Poisson process and an exponential distribution of the service time of each device according to the M/M/S queue model. Here, M/M/S represents a queue model with identical servers, where arrival is determined by a Poisson process and service time follows a negative exponential distribution [27]:

$$Obj: \min \ z_k \tag{3a}$$

$$\lambda_k^{RH} = \max \left\{ \lambda_{k,t} \right\} \tag{3b}$$

$$W_k^{RH} \leq W^{allowed} \quad \forall k \tag{3c}$$

$$W_k^{RH} = \frac{\left(z_k . \rho_k^{RH}\right)^{z_k} . \rho_k^{RH}}{\lambda_k^{RH} \left(z_k\right)! \left(1 - \rho_k^{RH}\right)^2} \pi_{0,k} \quad \forall k \tag{3d}$$

$$\pi_{0,k} = \left[\sum_{n=0}^{z_k - 1} \frac{\left(z_k \rho_k^{RH}\right)^n}{n!} + \frac{\left(z_k \rho_k^{RH}\right)^{z_k}}{\left(z_k\right)! \left(1 - \rho_k^{RH}\right)} \right]^{-1} \tag{3e}$$

$$\rho_k^{RH} = \frac{\lambda_k^{RH}}{z_k \mu} \tag{3f}$$

The objective function shown in (3a) is to minimize the number of fast charging nozzles required in the FCS. Equation (3c) shows that the average waiting time for charging during peak traffic hours should be within a predefined range. Equation (3d) explains how to calculate the waiting time in the queue theory. The probability that there are no vehicles under charging service in the FCS is represented by $\pi_{0,k}$. By increasing the number of charging devices, the waiting time in the queue can be reduced. In general, proper charging service and service facilitation can significantly increase the penetration rate of electric vehicles. However, on the other hand, increasing the number of charging devices increases the investment in the project, which is not economically feasible. Therefore, in order to optimally install charging stations, a criterion is used to consider the tolerance threshold of wallowed customers for charging at each charging station. In this way, if the customer's waiting time exceeds a certain time, the customer will leave the charging station. Since obtaining the inverse functions and the direct solution of the relationship is a complex task, a counting method is used to solve it. In this method, an initial value for the number of charging devices is assigned to the candidate locations of charging stations according to the maximum $\lambda_{k,t}$ in the time periods. In each iteration, one unit is added to the number of charging devices, and W^{RH} is calculated, and its value is compared with $W^{allowed}$. This continues until the average waiting time for charging is less than a certain value. The obtained value Z_k will be the economic number of charging devices. Once the size of each FCS is obtained, the total charging demand at each time can be calculated according to (4a) and (4b).

$$P_{k,t}^{FCS} = \rho_{k,t}.z_k.p^{FCS} \quad \forall k,t \tag{4a}$$

$$\rho_{k,t} = \frac{\lambda_{k,t}}{z_k.\mu} \quad \forall k,t \tag{4b}$$

C. Upper-Level Problem

The UL problem objective is to determine the location and capacity of FCS installations, aiming to maximize the profits of the private owner.

$$\underset{UL}{Maximize}\left(R^{FCS} - \mathrm{Cos}\,t^{FCS} - C^{inv}\right) \tag{5a}$$

$$R^{FCS} = 365.C^{sell}.\sum_{i\in I}\sum_{t\in T} U_i.P_{i,t}^{FCS} \tag{5b}$$

$$Cost^{FCS} = 365\sum_{t\in T}\sum_{k\in K} Cost_{i,t}^{up} \tag{5c}$$

$$C^{inv} = \frac{\varepsilon(1+\varepsilon)^{n_{FCS}}}{(1+\varepsilon)^{n_{FCS}}-1}\sum_k u_k\left(\alpha^{CHF} z_k + \alpha_k^{LS} z_k + \alpha_k^{F}\right) \tag{5d}$$

$$-U_i.bigM \le Cost_{i,t}^{up} \le U_i.bigM \tag{5e}$$

$$Cost_{i,t}^{up} \le \left(1-U_i\right).bigM + \lambda_{i,t}.P_{i,t}^{fcs} \tag{5f}$$

$$Cost_{i,t}^{up} \ge -\left(1-U_i\right).bigM + \lambda_{i,t}.P_{i,t}^{fcs} \tag{5g}$$

According to (5a), the objective function of the problem is to maximize the profit of the private owner, which includes the income from selling energy to electric vehicles, the cost of purchasing energy from the distribution network, and the cost of establishing FCSs, which are given in (5b) – (5c), respectively. According to (5e)– (5g), if an FCS is established, the cost of purchasing energy from the distribution network will be obtained by multiplying the purchased power by the hourly price of energy in the relevant bus.

It should be noted that the energy price per bus is the dual variable related to the constraint of equality of generated and consumed power per bus, which is obtained from the low-level problem.

D.Lower-Level Problem

The lower-level problem aims to minimize the energy production costs for the distribution network operator. For this purpose, DC optimal Power flow equations have been used, which are expressed in (6a)–(6g).

$$\underset{LL}{\text{Minimize}}\sum_t\sum_s \omega_s.(\sum_g C_g P_{g,t,s} + C_{t,s}^{grid}P_{i=1,t,s}^{grid} + \sum_i C^{ls}P_{i,t,s}^{ls}) \tag{6a}$$

$$\sum_{\square(g)=i} P_{g,t,s} + P_{i=1,t,s}^{grid} = P_{i,t,s}^{L} - P_{i,t,s}^{LS} + P_{i,t}^{FCS} + \sum_{s(l)=i} P_{l,t,s}^{flow} - \sum_{r(l)=i} P_{l,t,s}^{flow} \quad \forall i,t,s \quad :\lambda_{i,t,s} \tag{6b}$$

$$P_{l,t,s}^{flow} = B_l\left(\theta_{s(l),t,s} - \theta_{r(l),t,s}\right) \quad \forall l,t,s: \quad \rho_{l,t,s} \tag{6c}$$

$$-\overline{P_l} \le P_{l,t,s}^{flow} \le \overline{P_l} \quad \forall l,t,s \; : \mu_{l,t,s}^{-}, \mu_{l,t,s}^{+} \tag{6d}$$

$$p_{i=1,t,s}^{grid} \ge 0 \quad ;\delta_{i=1,t,s}^{1} \tag{6e}$$

$$0 \le P_{g,t,s} \le \overline{P_g} \quad \forall g, \forall t, \forall s \quad :\delta_{g,t,s}^{-}, \delta_{g,t,s}^{+} \tag{6f}$$

$$0 \le P_{i,t,s}^{LS} \le PD_{i,t,s} \quad \forall i,\forall t,\forall s: \; \eta_{i,t,s}^{-}, \eta_{i,t,s}^{+} \tag{6g}$$

The primal set of variables for each LL problem is $\Xi^{primal} = P_{i=1,t,s}^{grid}, P_{g,t,s}, \theta_{l,t,s}, P_{i,t,s}^{LS}$ while its dual set of variables is

$$\Xi^{dual} = \lambda_{tis}, \rho_{lts}, \mu_{lts}^{+}, \mu_{lts}^{-}, \delta_{ts}^{1}, \delta_{gts}^{+}, \delta_{gts}^{-}, \eta_{lts}^{+}, \eta_{lts}^{-}$$

The objective function of the low-level problem, which is to minimize the operating costs of the distribution network, is shown in (6a). This cost includes DG's energy production costs, the cost of purchasing energy from the upstream grid, and the load shedding cost. The equation (6b) ensures equality of generation and consumption power on each bus. The DC load flow is expressed in (6c).

The minimum and maximum power passing through each line is shown in (6d). The equation (6e) indicates that the distribution network is connected to the upstream grid via Bus 1. The network only receives energy from the upstream grid, and the possibility of selling energy back is not considered. The minimum and maximum generator capacities, as well as the curtailed load, are specified in the (6f)-(6g).

It is worth noting that the dual variable of each constraint is written in the same equation.

E. Transforming the Bi-Level Model to a Single-Level

If the LL problem is linear and convex, the bi-level model can be transformed into a single-level model using the KKT conditions, which introduce inherently non-linear complementary constraints. Since the proposed model's LL problem is linear and convex, the KKT conditions are applied to convert it into a single-level problem. This single-level linear optimization problem, known as a Mathematical Program with Equilibrium Constraints (MPEC), can then be solved using solvers such as CPLEX.

$$\underset{\Xi UL}{\textbf{Maximize}} \; (1a) \tag{7a}$$

$$\text{Subject to } (5b)\text{-}(5g),(6b),(6c) \tag{7b}$$

$$C_{t,s}^{grid} - \lambda_{i,t,s} - \delta_{i=1,t,s}^{1} = 0 \quad \forall i=1, \forall t, \forall s \tag{7c}$$

$$\lambda_{i=r(l),t,s} - \lambda_{i=s(l),t,s} + \rho_{l,t,s} - \mu_{l,t,s}^{-} + \mu_{l,t,s}^{+} = 0 \quad \forall l,t,s \tag{7d}$$

$$C^{ls} - \lambda_{i,t,s} - \eta_{i,t,s}^{-} + \eta_{i,t,s}^{+} = 0 \quad \forall i,t,s \tag{7f}$$

$$\rho_{r(i)=l,t,s} - \rho_{s(i)=l,t,s} = 0 \quad \forall i,t,s \tag{7g}$$

$$0 \le \left(\overline{P_l} - P_{l,t,s}^{flow}\right) \perp \mu_{l,t,s}^{+} \ge 0 \quad \forall l, \forall t, \forall s \tag{7h}$$

MPEC

Optimal siting /sizing of FCSs

Maximizing the profit of FCS invester (UL objective function)

Subject to:

Upper-level constraint

lower level constraint

Optimization constraint of KKT

complementary constraint of KKT

Fig. 2: The framework of the proposed model as MPEC

$$0 \le \left(P^{flow}_{l,t,s} + \overline{P_l}\right) \perp \mu^{-}_{l,t,s} \ge 0 \quad \forall l, \forall t, \forall s \qquad (7i)$$

$$0 \le p^{grid}_{i,t,s} \perp \delta^{1}_{i,t,s} \ge 0 \quad \forall i=1, \forall t, \forall s \qquad (7g)$$

$$0 \le P^{LS}_{i,t,s} \perp \eta^{-}_{i,t,s} \ge 0 \quad \forall i, \forall t, \forall s \qquad (7k)$$

$$0 \le \left(PD_{i,t,s} - P^{LS}_{i,t,s}\right) \perp \eta^{+}_{i,t,s} \ge 0 \quad \forall i, \forall t, \forall s \qquad (7l)$$

$$0 \le P_{g,t,s} \perp \delta^{-}_{g,t,s} \ge 0 \quad \forall g, \forall t, \forall s \qquad (7m)$$

$$0 \le \left(\overline{P_g} - P_{g,t,s}\right) \perp \delta^{+}_{g,t,s} \ge 0 \quad \forall g, \forall t, \forall s \qquad (7n)$$

$$0 \le P^{LS}_{i,t,s} \perp \eta^{-}_{i,t,s} \ge 0 \quad \forall i, \forall t, \forall s \qquad (7o)$$

$$0 \le PD_{i,t,s} - P^{LS}_{i,t,s} \perp \eta^{+}_{i,t,s} \ge 0 \quad \forall i, \forall t, \forall s \qquad (7p)$$

Eq. (7a) shows that the MPEC model objective function is the same as the UL problem function. Constraint (7b) contains the UL constraints and the equality constraints included in the LL problems. Equalities (7c)–(7g) and the complementarity conditions (7h)–(7p) are the KKT optimality conditions of the LL problems.

F. MPEC Linearization

The MPEC single-level model is a non-linear problem because of complementary constraints, in (7h)–(7p). Because the presence of non-linear complementary constraints makes the obtained single-level model non-linear, the suggested model is linearized using a technique based on auxiliary binary variables and suitably large integers. For example, linearization of 0 ⩽ a ⊥ b ⩾ 0 is (8): [28]

$$\begin{cases} 0 \le a \le U.M \\ 0 \le b \le (1-U).M \\ U \in [0,1] \end{cases} \qquad (8)$$

Note that the variables of the resulting MILP are those included in the set, as well as the auxiliary binary variables used for the linearization of the complementarity conditions. The framework of the proposed model as MPEC is illustrated in Fig. 2.

III. CASE STUDY

To implement the proposed concepts, the IEEE 33-bus system [29] (Fig. 3) and the transportation network presented in [30] (Fig. 4) have been used. The transportation network includes 24 traffic nodes and 21 Sioux Falls routes. This network consists of 76 paths, 24 nodes, and 552 origin-destination pairs. The loads at various buses of the distribution network follow a 24-hour load profile as shown in Fig. 5. The network operates at a voltage level of 12.66 kV and is fed from the substation located at bus 1. The maximum power passing through the lines is assumed to be 3000 kW. Additional information about this network can be found in [30]. Candidate locations for installing FCS and the investment costs associated with each location are provided in Table I.

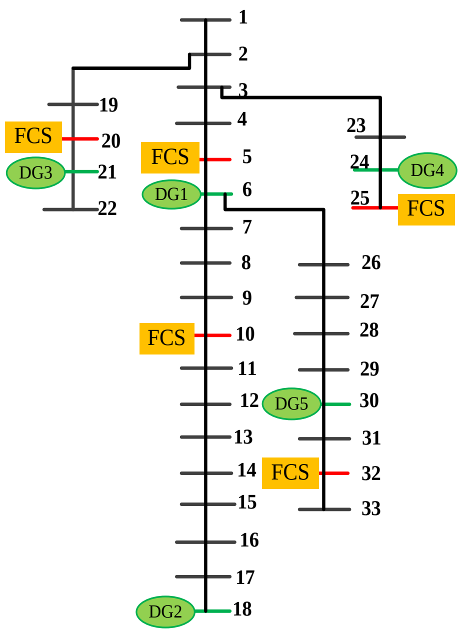

Fig. 3: IEEE 33-bus electrical network

TABLE I
Construction Costs of FCS

Candidate FCS	1	2	3	4	5
location	12[a] (5)[b]	3[a] (10)[b]	10[a] (20)[b]	15[a] (25)[b]	18[a] (32)[b]
$\alpha^{CHF}(10^4\$)$	2.35	2.35	2.35	2.35	2.35
$\alpha^{LS}(10^4\$)$	1.017	1.068	0.814	0.916	1.017
$\alpha^{k}(10^4\$)$	16.3	16.3	16.3	16.3	16.3

[a] node number in transportation network
[a] node number in electrical network

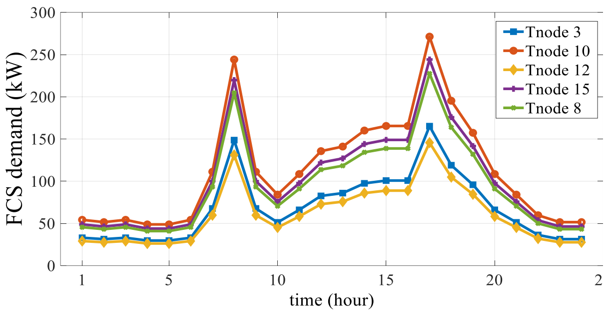

Fig. 6 : FCS hourly demand

Considering the candidate locations for installing FCSs and using the user equilibrium-based Traffic assignment and queuing theory models, the power demand of each candidate FCS is shown in Fig. 6. The electricity selling price to electric vehicles, which is one of the key factors influencing the charging station owner's decision, is also shown in Fig. 7 .Other parameters required to implement the model are also given in the Table II.

TABLE II
Settings of Some Crucial Parameters

parameter	value	parameter	value
Z_{min}	6	Z_{max}	10
$W_{allowed}$	5 min	n_{FCS}	5
ε	10%	C_{LS}	2.5 \$/kwh
B_{sell}	1.35 \$/kwh		

TABLE III
Summary of Simulation Results

parameter	value
Optimal installation location	**[15,18]**
Charger number	**[9,7]**
FCS Owner's profit	**6,147 $**
Investment cost	**139,255 $**
FCS cost	**2,183,639 $**
FCS revenue	**2,329,041 $**

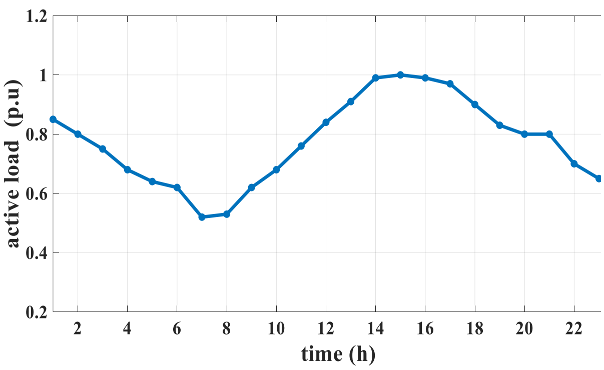

Fig.5: Load profile

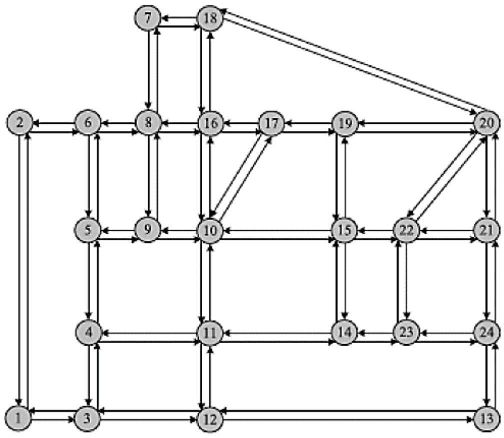

Fig. 4: Sioux Falls transportation network

A. Simulation and results analysis

The proposed MILP bi-level model was implemented in the GAMS software, and with the CPLEX solver, the results of which are given in Table III.

Analyzing the results, the FCS owner will establish two FCS at traffic nodes 15 and 18 (25 and 32 of the electrical network) with 9 and 7 chargers, respectively. By establishing these two stations, the station owner will earn a profit of $6,174, of which 139,255 $ will be spent on establishing the station and $2,183,369 on purchasing energy from the distribution network. There will also be an income of $2,329,041 from selling energy to electric vehicles.

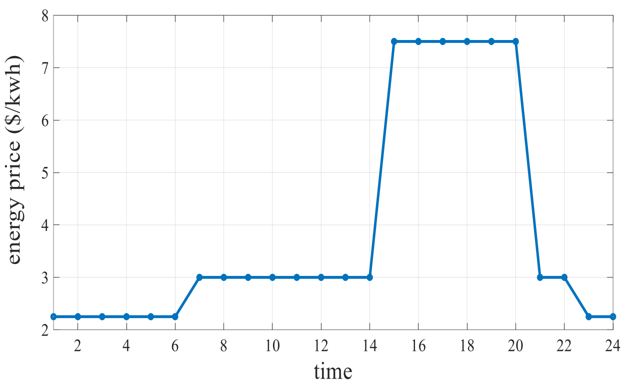

Fig. 7: Price of purchasing energy from the upstream network during the day

In this case, the cost to the DSO is 105,934,300 $. In this case, the cost of generating energy by DGs, the cost of purchasing energy from the upstream network, and the cost of LS in one day are 31,496 $, 246,440 $, and 12,293 $, respectively. Given the presence of three DGs in the distribution network, the active power generated by each DG is shown in Fig. 8

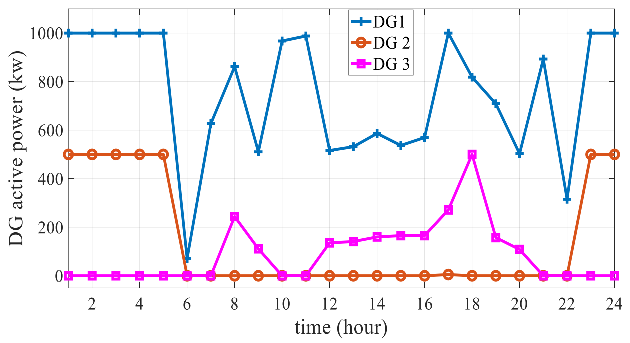

Fig. 8: DG active power

Fig. 9 illustrates the temporal and spatial variations of the nodal electricity prices across different buses over a 24-hour period. As shown, the LMP values are generally low and uniform during off-peak hours, indicating balanced power flow and low network congestion. However, during hours 16–19, a significant increase in the LMP is observed at several buses (particularly around buses 5 and 20), reflecting higher demand and possible local congestion in the distribution feeders. These higher nodal prices are directly linked to the power balance constraints in the lower-level optimization, where dual variables represent the marginal cost of supplying an Additional unit of power. These LMPs are used as the reference prices for energy transactions between the DSO and the FCS owners in the bi-level framework.

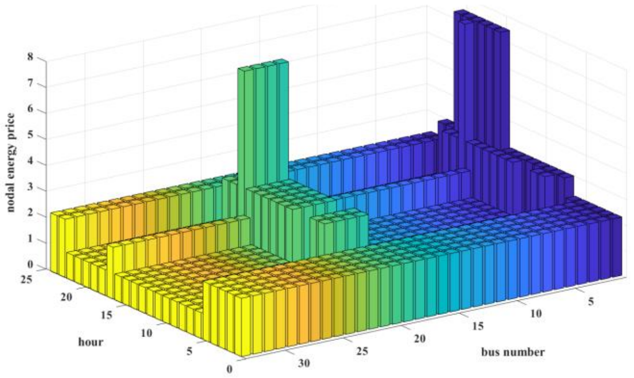

Fig. 9: nodal energy price

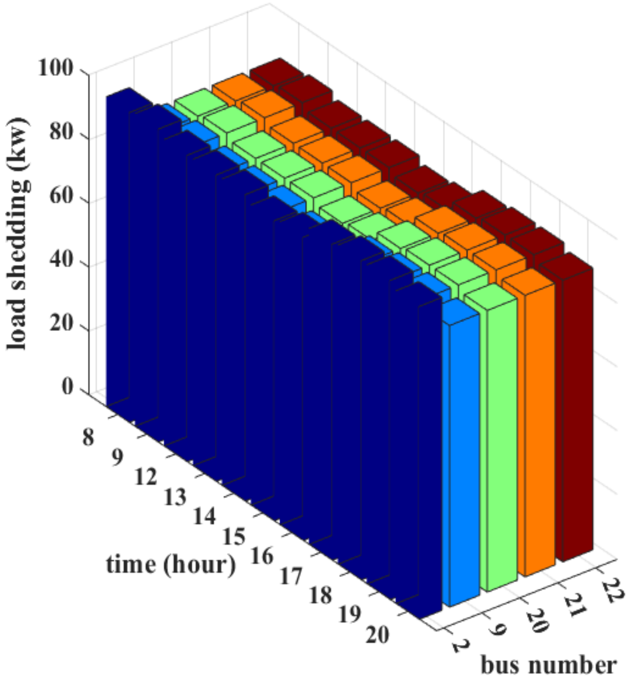

Fig. 10: Load shedding

TABLE IV
Optimal Locations and Number of FCSs Under Different Electricity Selling Prices to EV Drivers

Price ($/kWh)	profit	Bus number	Chargers number
1.35	6,146	25,32	9,7
1.36	23,398	25,32	9,7
1.37	40,650	25,32	9,7
1.38	59,390	5,25,32	6,9,7
1.39	86,037	5,10,25,32	6,8,9,7

The amount of LS on each bus over 24 hours is also shown in Fig. 10. As illustrated in the figure, during the early hours before 8:00, when the network load is relatively low, no load shedding occurs. However, as the demand increases after this period, certain buses experience load curtailment to maintain system stability and prevent overloading conditions.

B. Sensitive analysis:

To assess the robustness of the proposed model, a sensitivity analysis is performed on the electricity selling price to EVs, which directly impacts the profitability and, consequently, the location and capacity decisions of the charging stations.

As shown in Table IV, the location and number of FCSs change with variations in the electricity selling price to electric vehicles. At lower prices (1.35–1.37 $/kWh), the optimizer selects buses 25 and 32 as the most profitable locations, each with 9 and 7 chargers, respectively. In this range, the profit gradually increases with the selling price, while the optimal sites remain unchanged. When the price increases to 1.38 $/kWh, an additional station is installed at bus 5, indicating that higher revenues justify expanding the charging infrastructure. Finally, at 1.39 $/kWh, another station appears at bus 10, leading to a network of four charging stations and the highest total profit. This trend shows that as the selling price rises, the profitability of the investment improves, encouraging the deployment of more FCSs in additional locations across the network.

IV. CONCLUSION AND FUTURE WORK

In this study, a bi-level optimization framework is developed for the siting and sizing of fast-charging stations, considering the interaction between the distribution system operator and the FCS investor. The lower level minimizes network operational costs via a DC optimal power flow, generating locational marginal prices that are passed to the upper level. The upper level maximizes the investor's profit by determining optimal FCS locations and capacities based on electricity prices and charging demand, modeled through a user-equilibrium traffic assignment and M/M/S queuing theory. The model is reformulated as a mixed-integer linear program using Karush–Kuhn–Tucker conditions and solved in GAMS. Simulation results on the IEEE 33-bus distribution system coupled with a 25-node transportation network demonstrate that the proposed approach identifies two optimal FCS locations with 9 and 7 chargers, resulting in a net daily profit of $6,147 for the investor, while maintaining feasible network operation. The derived locational marginal prices vary spatially and temporally, directly influencing investment decisions. Sensitivity analysis indicates that higher electricity selling prices shift optimal locations and increase investor profit.

In this study, the power demand at each candidate location for establishing FCSs was predefined. The selection or non-selection of a candidate site does not affect the charging demand of other stations. However, in reality, part of the charging demand from nearby stations may shift to the newly established ones. The dynamic behavior of charging demand among stations can play a significant role in the investor's siting decisions as well as in satisfying the distribution network constraints. However, considering this dynamic behavior would make the cost of purchased energy from the DSO nonlinear, preventing the use of conventional solvers such as CPLEX to solve the problem.

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CONFLICT OF INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

AUTHORS' CONTRIBUTIONS

Mohammad Alizadeh: Conceptualization, Methodology, Investigation, Software. Ali godarzi amlashi: Supervision, Validation, Writing – review & editing.

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