

# Torque Profile Optimization in a Three-Phase 12 by 8 Switched Reluctance Motor by Using Genetic Algorithm in the Gate Pulse Generation

Mehran Rafiee, Hamid Reza Izadfar\*, Yousef Alinejad Beromi

**Abstract--** Despite a large number of advantages, Torque Ripple (TR) is the most important drawback of Switched Reluctance Motor (SRM). In the presented study, TR is reduced by optimizing the value of the gate pulse angle which plays a leading role in the generated torque profile of SRM. For the Optimization, one of the best strategies of the Genetic Algorithm (GA) which was named Non-dominated Sorting Genetic Algorithm II (NSGA-II) is used. Its duty was to detect the best solution (gate pulse angle) which forces the machine to generate the highest value of torque with the least rate of TR. The proposed control algorithm was run in a simulation process of a three-phase 12 by 8 SRM. Then, the statistical results were compared with the results of a custom GA and the traditional control method of SRMs. The comparison proves that by using the presented algorithm, not only the generated TR of the selected SRM is significantly reduced by roughly 210%, but also the generated torque profile of the machine is improved as well. In addition, the presented method is a low-cost strategy with less complication in comparison with other similar torque profile correction techniques.

**Keywords:** Genetic Algorithm, SRM, TR, Torque.

## NOMENCLATURE

A nomenclature list, if needed, should precede the Introduction.

## I. INTRODUCTION

THE SRM has a lot of merits which are mainly resulted from its simple structure. However, its non-linear behavior, such as the triangular inductance profile, creates some problems and challenges in the SRM control and operation. TR is the most important problem in these types of machines which is the main source of vibration and acoustic noise generation. As is seen in Fig.1, different methods have been presented so far to reduce TR and correct the torque profile in SRMs. Generally, they can be classified into three types:

- ✓ Changing the motor structure.
- ✓ Software methods.
- ✓ Changing the converter structure.

Typically, “Changing the motor structure” and “Software” methods are very expensive. “Software” methods use some variables such as current, torque, rotor position, etc. as feedback parameters; hence, they require a large number of calculations and complex programming. They require a powerful processor with a high operating frequency and a large memory increasing the cost of the controller board.

Each of these methods is divided into some strategies which are explained as follows:

In [1], a two-layer SRM is designed, and TR is reduced by changing the geometry shape of the motor. The rotors of layers are shifted some degrees and are not in the same direction which dramatically decreases the effect of TR.

In [2] and [3], it is proven that increasing motor poles and choosing the appropriate quantity for dimensions of stator and rotor significantly improve the generated torque profile of SRM.

In [4-8], different types of multi-layer SRMs are presented. In these types of motors, each layer is energized for a short period which causes the phase to have less TR. In addition, these types of motors have some other advantages, as an illustration, thanks to independent layers, there is no isolation problem in multi-layer SRMs. Moreover, motor cooling can be easily done. Above all, there is more area for coil wrapping in these types which means the motor can generate more amount of torque.

In [9-12], the Torque Sharing Function (TSF) method is presented which reduces TR by changing the control algorithms of the gate pulse generation of SRM. The TSF method uses the Torque - Rotor position - Current relation of SRM. The relation can be applied in the linear or non-linear form such as exponential, quadratic or sinusoidal forms. In this method, the (linear or non-linear) relation is registered in the memory of the microprocessor which generates gate pulses. The generated torque and rotor position of SRM are sampled with a high frequency during the motor operation by the microprocessor. Afterward, the best gate pulses are generated by the microprocessor according to the TSF profile of the selected SRM to generate the most appropriate torque

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profile.

Another TR reduction method is based on “Changing the converter structure” such as using a multilevel converter. In this strategy, the generated torque of SRM is calculated by sampling the motor phase current, rotor position, and speed. Then, the proper voltage level for the phase, by which the current profile is improved, is detected by the controller. According to the value of the detected voltage, suitable gate pulses are applied to the transistors of the converter to supply the phase winding by the detected voltage. By correcting the current profile, the torque profile is corrected [13], [14]. Note that, although “Changing the converter structure” is the cheapest strategy in comparison with “Changing the motor structure” and “Software” methods, multilevel converter requires a powerful controller and can be classified as a “Software” method as well.

The “Software” methods can be optimized by some intelligent algorithms such as fuzzy, neural network, etc. [15], [16]. Lagging-leading the phase fire (gate pulse) angle is another “Software” method as well [17].

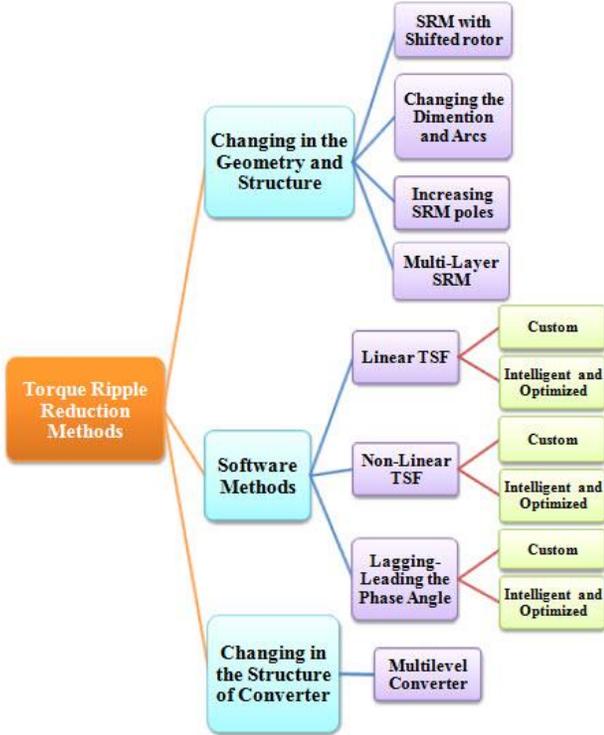


Fig. 1. Different types of TR reduction methods

In this paper, the mathematical model of a three-phase 12 by 8 SRM is designed in software using the specifications of a real motor. Then, NSGA-II [18] is used to simulate the motor model in the desired speed which is determined by the user. The proposed method estimates the optimized and the best gate pulse angle which forces the motor to achieve the highest level of the generated torque with the least percentage of TR on its shaft.

NSGA-II has some advantages in comparison with custom GA. One of the specifications of the NSGA-II strategy is that the populations of parent and offspring are combined and the

best population is selected according to the fitness and spread. Hence, NSGA-II can detect much better solutions with better convergence. In addition, this method increases elitism which can prevent loss of appropriate solutions during operation. Therefore, it is significantly able to speed up the performance of the algorithm, as is proven in [19] and [20].

According to Fig.1, the proposed algorithm of this study is classified as a “Software” method for TR reduction. As was mentioned, the main drawback of “Software” methods is that their programming, computational and total costs are very high generally (especially in TSF methods) and they need an expensive powerful processor chip. On the other hand, the proposed NSGA-II technique does not require such a processor. The optimized results which are obtained from the algorithm for each range of speed can be easily placed in a look-up table in the memory of a low-cost microcontroller to drive SRM appropriately. This study is organized as follows: Section two studies the origin of generating TR in SRMs. In the third section, NSGA-II and its important details are explained. The fourth section introduces the proposed control algorithm and the specifications of the selected SRM. The simulation results of the presented NSGA-II, custom GA, and conventional methods are declared in the fifth section. In Section Six, the results obtained in the simulations are compared with each other. In Section seven, the comparison of the presented NSGA-II method with some other techniques is declared; then, the paper is followed by a conclusion in Section eight.

## II. THE ORIGIN OF TR

The stator and rotor poles of SRM are salient; hence, the air gap between the stator and rotor poles has different values in each position of the rotor. When the rotor pole is in an unaligned position concerning the stator pole, the maximum air gap exists between the poles. On the contrary, in a fully aligned position, the air gap between the stator and rotor poles has the least value. This intrinsic characteristic of SRM results in the reluctance ( $R$ ) profile to be triangular as is illustrated in Fig. 2. Accordingly, the inductance profile is triangular as well (Since  $L = \frac{(\text{Number of Windings})^2}{R}$ ).

Moreover, the electrically generated torque of a phase in an unsaturated SRM can be calculated as follows:

$$T_{ph} = \frac{i^2}{2} \cdot \frac{dL(i, \theta)}{d\theta} \quad (1)$$

Where  $\theta$  is the gate pulse angle in rad and  $i$  is the current of the phase in Ampere. As is seen, the generated torque is independent of the sign of the current and its sign only depends on the derivative  $\frac{dL(i, \theta)}{d\theta}$ . In the motor mode operation of SRM, positive torque must be generated to rotate the rotor which means phases should be energized when  $\frac{dL(i, \theta)}{d\theta} > 0$ . Therefore, as is shown in the gate pulse profile of a phase in Fig. 2, the phase is switched on between the start of alignment

and fully aligned positions of the rotor. Accordingly, a positive reluctance torque is generated which forces the stator and rotor poles to be aligned.

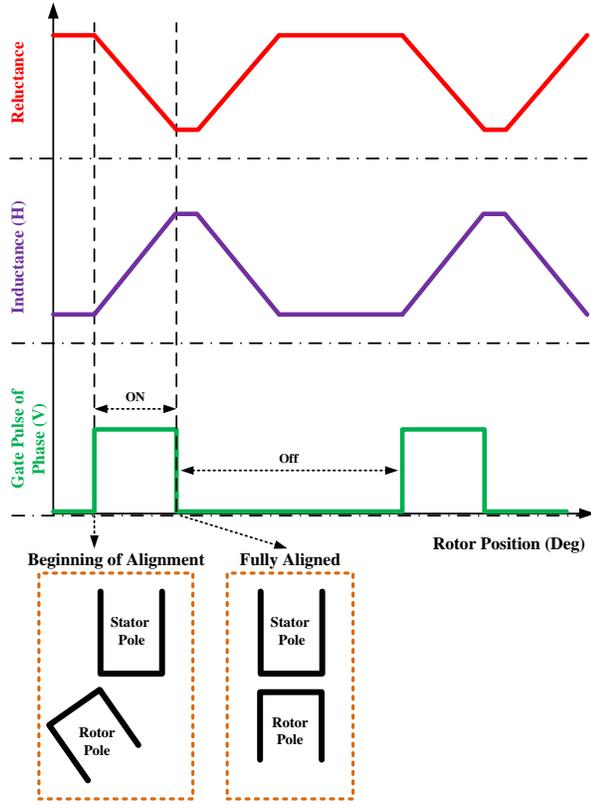


Fig. 1. Profiles of reluctance, inductance, and gate pulse in each phase of a three-phase SRM

Negative torque generation in the motor mode operation produces TR which is the main source of vibration and the acoustic noise creation in SRM. Two main reasons can result in negative torque generation in SRMs:

- ✓ Energizing the phase when  $\frac{dL(i,\theta)}{d\theta} < 0$

According to Equation (1), if the phase winding is excited after a fully aligned position, where  $\frac{dL(i,\theta)}{d\theta} < 0$ , the generated torque will be negative. This means that the gate pulse angle should be generated with high accuracy.

- ✓ The energy which is stored in the phase:

As is shown in Fig. 3, when the SRM phase is switched off (for instance, in the fully aligned position of the rotor), the value of the phase inductance is at the highest level. In this case, according to Equation (2), which is the dynamic mathematical model of an unsaturated SRM, some energy is stored in the winding which means the phase contains some amount of current even after switching off. This stored current; which is known as Current Tail (CT), generates negative torque according to Equation (1).

ON:  $V = V_{ON} \rightarrow$

$$\left[ V - r i_{on} + \omega i_{on} \frac{dL(i_{on}, \theta)}{d\theta} + L(i_{on}, \theta) \frac{di_{on}}{dt} = 0 \right]$$

OFF:  $V = 0 \rightarrow$

$$\left[ V - r i_{off} + \omega i_{off} \frac{dL(i_{off}, \theta)}{d\theta} + L(i_{off}, \theta) \frac{di_{off}}{dt} = 0 \right] \quad (2)$$

In Equation (2),  $r$  is the winding resistance in Ohm,  $\omega = \frac{d\theta}{dt}$  is the angular velocity of the rotor shaft in rad/sec. Note that, in this equation, the terms from left to right are respectively the phase applied voltage ( $V$ ), resistive voltage drop, induced Electro-Motive Force (EMF) voltage drop, and inductive voltage drop. Notice that, the first-mentioned reason may occur by applying a gate pulse with an inappropriate angle. For instance, a gate pulse by which the phase is switched-off after the fully aligned position, where  $\frac{dL(i,\theta)}{d\theta} < 0$ . Therefore, negative torque and TR are generated.

The second reason is due to the inherent characteristic of SRM. Actually, in this case, the phase is switched off when  $\frac{dL(i,\theta)}{d\theta} > 0$ ; however, the value of the gate pulse angle is very high which means the phase is switched off in a closed position concerning the fully aligned position. In other words, in this case, when the phase is switched off, the inductance value is very high. Hence, the value of CT is very high as well; accordingly, a tremendous amount of negative torque and TR are generated.

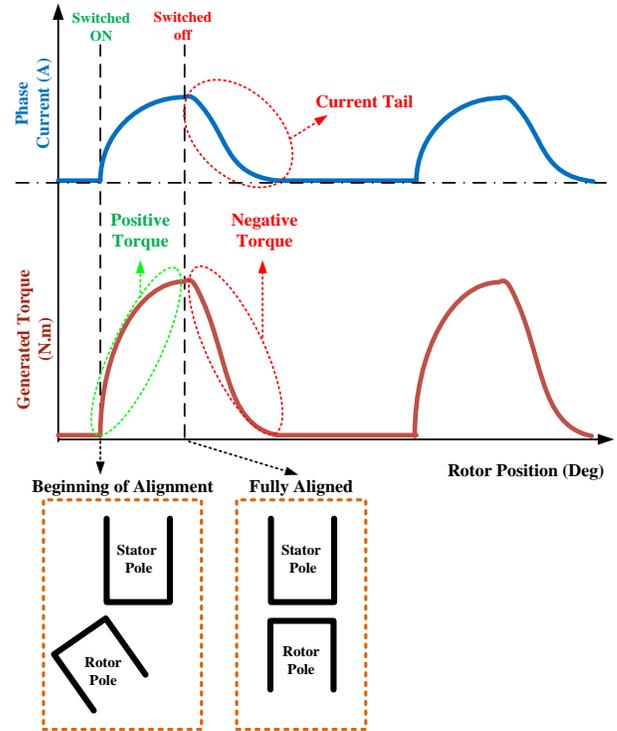


Fig. 3. Profiles of current and torque in each phase of a three-phase SRM

If we assume that the SRM magnetic model is linear and there is no mutual inductance between the phases, the total electrical torque, which is shown in Fig. 4, can be obtained as follows:

$$T = \sum T_{ph} = \sum \frac{i^2}{2} \cdot \frac{dL(i,\theta)}{d\theta} \quad (3)$$

According to Equation (3), CT produces some negative torque (acts as a brake) and makes SRM generate triangular

(or sawtooth) total torque. In other words, the motor generates TR. Note that torque ripple is calculated as follows:

$$TR = \frac{T_{\text{peak}} - T_{\text{bottom}}}{T_{\text{avg}}} \times 100\% \quad (4)$$

Where,  $T_{\text{peak}}$ ,  $T_{\text{bottom}}$ , and  $T_{\text{ave}}$  are respectively maximum, the minimum, and average value of the motor generated torque. The mechanical equation of SRM can be written as follows:

$$T = T_L + T_m = T_L + J \frac{d\omega}{dt} + B\omega \quad (5)$$

Where  $T_L$  is the load torque,  $T_m$  is the mechanical torque,  $B$  is the friction coefficient and  $J$  is the moment of inertia. Equation (5) proves that TR significantly influences the generated mechanical torque of SRM. It shows itself as acoustic noise and vibration which are known as important problems in SRMs.

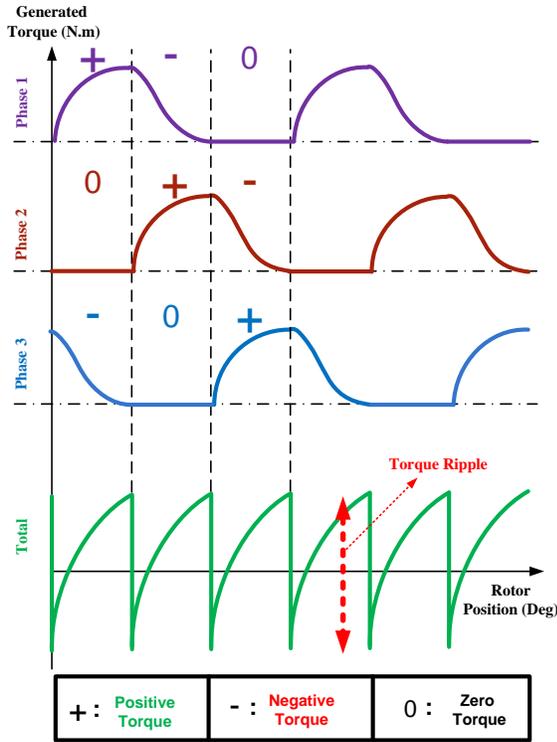


Fig. 4. Profiles of generated torque in SRM in a three-phase motor

### III. SPECIFICATION OF NSGA-II

NSGA-II algorithm is introduced as the modified version of the NSGA method to reduce some difficulties it. When the algorithm is initialized, a random parent population ( $P_i$ ) with  $N$  chromosomes is created and sorted according to the non-domination level. After this stage, we have some non-dominated sets ( $F_1, F_2$ , etc.) so that the best solutions are respectively  $F_1, F_2$ , and so on. Then, the offspring population ( $O_i$ ) with  $N$  chromosomes is created using the three basic steps of GA which are natural selection, recombination, and mutation.

After initialization, different stages of the NSGA-II are performed, as is shown in Fig. 5. The parent and offspring chromosomes ( $P_i$  and  $O_i$ ) are combined in the first step which comprises a new population ( $P_{i+1}$ ) with  $2 \times N$  chromosomes. The members of  $P_{i+1}$  are sorted according to the non-domination level to create the non-dominated sets ( $F_1, F_2$ , etc.) again. Now, we should exactly select  $N$  members from the best solutions. Some of the non-dominated sets may be completely selected; moreover, the remaining members should be selected from the subsequent non-dominated set by using crowding distance sorting. The other members are rejected and a new population ( $P_{i+1}$ ) is obtained with  $N$  chromosomes whose members are the best solutions of the former generation. The chromosomes with the best fitness are preserved for the next generation in this algorithm. Then, the offspring population ( $O_{i+1}$ ) is created by using natural selection, recombination, and mutation, and the procedure is repeated in the next generation ( $i+2$ ).

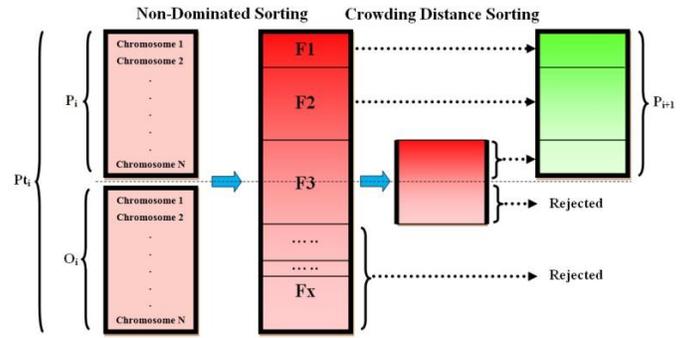


Fig. 5. Different steps of the NSGA-II [18]

### IV. THE PROPOSED CONTROL ALGORITHM

In this paper, in each range of the SRM speed, NSGA-II obtains the optimum value of the gate pulse angle by two objectives functions which are minimizing TR as is given in equation (6) and maximizing the generated torque of SRM which is declared in Equation (7):

$$f_1 = \text{Min} (TR) = \text{Min} \left( \frac{T_{\text{peak}} - T_{\text{bottom}}}{T_{\text{avg}}} \times 100\% \right) \quad (6)$$

$$f_2 = \text{Max} (T) = \text{Max} \left( \sum \frac{i^2}{2} \cdot \frac{dL(i,\theta)}{d\theta} \right) \quad (7)$$

Note that, by increasing the value of the gate pulse angle in SRM, both values of the generated torque and TR increase. As shown in Section II, the value of CT incredibly surges when the gate pulse angle is very high (each phase is switched off near the fully aligned region where the value of inductance is very high). In other words, although a higher value of the gate pulse angle is desired (because it results in torque increase), a very high gate pulse angle surges the generated TR. Accordingly, the goal of the proposed control algorithm is to detect the best value of the gate pulse angle (solution) which generates the highest value of torque and the least value of

TR. In addition, for the presented paper in which a three-phase 12 by 8 SRM is used, the positive torque region of the motor (where  $\frac{dL(i,\theta)}{d\theta} > 0$ ) starts at  $0^\circ$  and ends at  $15^\circ$ . If the gate pulse angle is more than  $15^\circ$ , braking torque will be generated (because in that region:  $\frac{dL(i,\theta)}{d\theta} < 0$ ). This note can be mentioned as a constraint; as a result, the upper and lower bound of the gate pulse angle is defined for the system as follows:

$$\theta_{deg} \in (0^\circ, 15^\circ] \quad (8)$$

Where  $\theta_{deg}$  is the real value of the gate pulse angle (operating parameter) in degree. The operating parameter is introduced for the system as a binary chromosome with eight bits. Hence, the equation of converting the binary chromosome to the real value is:

$$\theta_{deg} = \frac{15 - 0}{2^8 - 0} \times d = \frac{3}{51} \times d \quad (9)$$

Where  $d$  is the decimal value of the binary chromosome. On the whole, the primary population size of the proposed NSGA-II is ten random chromosomes each of which has eight bits (gate pulse angles). Middle single-point cross-over is used in the recombination process with the crossover probability ( $P_c$ ) of 1. Furthermore, the mutation probability ( $P_m$ ) of the algorithm is 0.1. The fitness function of the generation “ $i$ ” of the population is considered as:

$$F_P^i = F_{TR}^i + F_T^i \quad (10)$$

$F_{TR}^i$  and  $F_T^i$  are respectively the TR fitness function and the generated torque fitness function of the generation “ $i$ ”. They are calculated as follows:

$$F_{TR}^i = \sum_{j=1}^N \frac{\text{Min}(TR_j^i)}{TR_j^i} \quad (11)$$

$$F_T^i = \sum_{j=1}^N \frac{T_j^i}{\sum_{j=1}^N T_j^i} \quad (12)$$

In the above equations,  $N$  is the population size and  $j$  is the chromosome number. The expression “ $\text{Min}(TR_j^i)$ ” detects the minimum value of TR among the TR values produced by the gate pulse angles of the generation “ $i$ ”.  $TR_j^i$  and  $T_j^i$  are respectively the values of TR and torque which are generated when the chromosome “ $j$ ” of the generation “ $i$ ” is used as the gate pulse angle of the SRM. As is seen in Equation (11), the less the TR values of a generation, the more the  $F_{TR}^i$  value of that generation. Additionally, Equation (12) shows that if the amounts of the generated torque of a generation are higher, the value of  $F_T^i$  will be higher for that generation as well. By using Equations (10) to (12), equation (13) is achieved as the final fitness function of the generation “ $i$ ”:

$$F_P^i = \sum_{j=1}^N \frac{\text{Min}(TR_j^i)}{TR_j^i} + \sum_{j=1}^N \frac{T_j^i}{\sum_{j=1}^N T_j^i} \quad (13)$$

Figure 6 illustrates the block diagram of the presented control algorithm. In the first step, SRM starts rotating based on the desired speed which is defined by the user. The desired speed determines the gate pulse frequency as follows:

$$f = \frac{S}{60 \times n_e} \quad (14)$$

Where,  $S$  is the motor speed in rpm and  $n_e$  is the number of the rotor rotation in each electrical period; for instance, it is  $\frac{1}{8}$  for a three-phase 12 by 8 SRM (In a three-phase 12 by 8 SRM: Mechanical period =  $8 \times$  Electrical period.). Then, the proposed NSGA-II starts to detect the best gate pulse angle according to the value of torque and TR which are calculated by using the current of the phases, rotor position, Equation (3), and Equation (4). In each generation stage of NSGA-II, the PWM generator unit produces gate pulses based on the detected gate pulse angle and the speed of the motor (frequency). Afterward, the converter drives the SRM by using the gate pulses. Then, the resulted values of the phase current and rotor position are used for the torque and TR calculation again. The process is repeated until the best solution (gate pulse angle) is achieved for the determined speed.

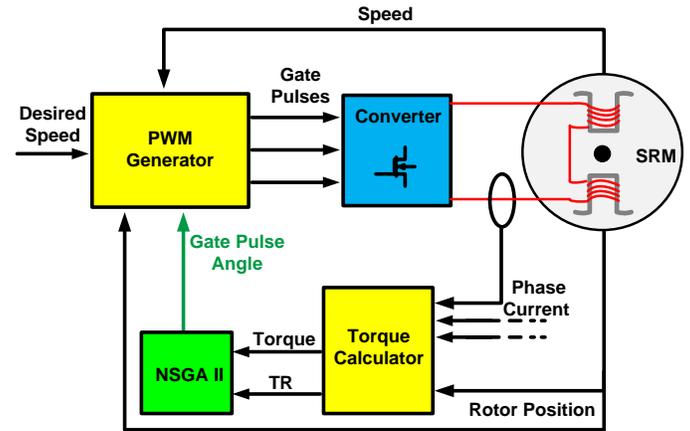


Fig. 6. The block diagram of the proposed control algorithm

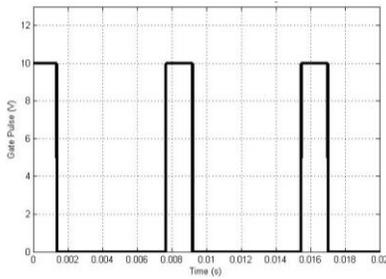
## V. SIMULATION RESULTS

As mentioned before, in this paper, a three-phase 12 by 8 SRM is selected for simulation. The general specifications of the selected SRM are presented in Table 1.

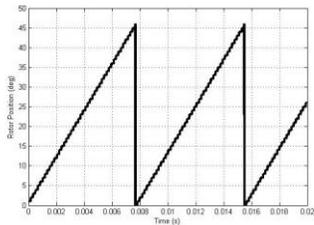
All the simulations are done in no-load conditions on an Intel Core 2 due to CPU @ 2.66 GHz with 4 GB RAM. In the first stage, the SRM is simulated by a conventional method with an  $8^\circ$  gate pulse angle and at the speed of 1000 rpm. After determining the desired speed for the system, the simulation is run for 0.02 sec. Figure 7 illustrates the gate pulse of one phase and rotor position of the SRM at the speed of 1000 rpm.

Table 1  
The specifications of the selected 12 by 8 three-phase SRM

Nominal power	50 watts
Nominal Voltage	12 volts
Nominal Speed	3000 rpm
Stator core outer diameter	60mm
Stator core inner diameter	52mm
Stator pole arc	15°
Air gap	0.25mm
Rotor core outer diameter	35mm
Rotor shaft diameter	8mm
Rotor pole arc	16°
Number of turns per pole	100
Phase Resistance	3.6 Ω
Maximum Inductance (in fully aligned)	10.1 mH
Minimum Inductance (in un-aligned)	3.6 mH



(a)

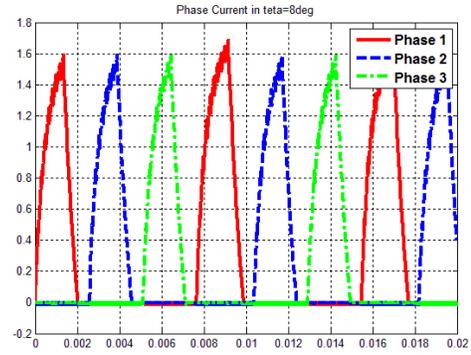


(b)

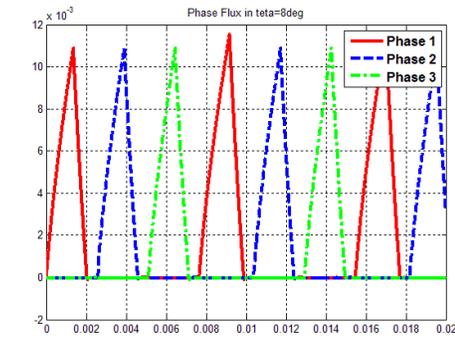
Fig. 7. (a): Gate pulse of one phase and (b): Rotor position of the motor in the speed of 1000 rpm in  $\theta = 8^\circ$

Figure 8 shows the current and flux waveforms of all phases at the speed of 1000 rpm. As is seen, by using an  $8^\circ$  gate pulse angle, the total current of the SRM is discontinuous (it is zero sometimes). The current profile results in the torque profile to be in the form of Fig. 9, which shows that the SRM generates a high value of TR.

Table 2 presents the statistical results which are obtained in this simulation. The maximum amount of torque is 2.8586 N.m and the minimum value is zero which means the generated torque of the SRM is discontinues. The concept of having this profile of torque is that the motor significantly generates vibration and acoustic noise in operation. The average value of the generated torque is 0.9723; therefore, by using Equation (4), the calculated TR is 294% which is very high.



(a)



(b)

Fig. 8. (a): Current and (b): Flux waveforms in all phases in the speed of 1000 rpm in  $\theta = 8^\circ$

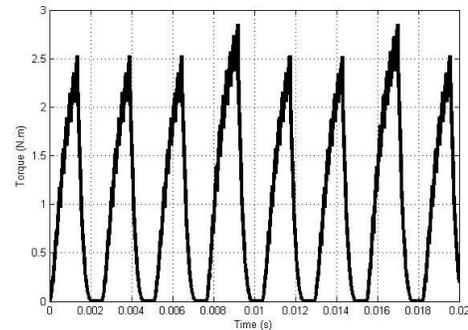


Fig. 9. Total generated torque of the SRM in the speed of 1000 rpm in  $\theta = 8^\circ$

Table 2  
The simulation results in the speed of 1000 rpm in  $\theta = 8^\circ$

Default Custom $\theta$ (deg)	TR (%)	$T_{peak}$ (N.m)	$T_{bottom}$ (N.m)	$T_{ave}$ (N.m)
8	294.00	2.8586	0	0.9723

In the next step, the SRM is simulated by using a custom method of GA in which three conventional steps of GA (natural selection, recombination, and mutation) are used. The population size is ten again. All recombination is implemented by middle one-point cross-over with unity probability. Furthermore, the mutation probability is 0.1. Hence, the conditions in the used GA are completely similar to those used in the selected NSGA-II. Moreover, the desired speed in the simulation is 1000 rpm again.

Table 3 illustrates the best, worst and average values of torque achieved from the final solutions (population) of the GA after optimization. Note that, these results are achieved after 51 iterations; additionally, the optimization time is 364.6791s. The maximum amount of torque is 3.0145 N.m obtained in 13.4118° gate pulse angle. Moreover, the minimum torque is 0.891 N.m, with a gate pulse angle of 1.5294°. In addition, the average torque of the last population is 2.8576 N.m.

Table 3

The best, worst and average values of torque in the last population of custom GA

Best Toque		Worst Toque		Average Toque
Value (N.m)	$\theta$ (deg)	Value (N.m)	$\theta$ (deg)	Value (N.m)
3.0145	13.4118	0.891	1.5294	2.8576

Table 4 reports the best and the worst TR values of the last generation. As seen, the maximum TR is 204.46% in 10° gate pulse angle. In addition, the best TR value is again achieved in 13.4118° gate pulse angle with roughly 103.44%.

Table 4

The best and worst values of TR in the last population of custom GA

	TR Value (%)	$\theta$ (deg)	$T_{peak}$ (N.m)	$T_{bottom}$ (N.m)	$T_{ave}$ (N.m)
Worst TR	204.46	10	2.7793	0	1.3594
Best TR	103.44	13.4118	3.0145	0.8531	2.0897

The goal is to obtain the best solution (gate pulse angle) to force the SRM to generate the highest value of torque with the least rate of TR. Hence, as is seen in Table 5, the best value for torque and TR are achieved in 13.4118° angle.

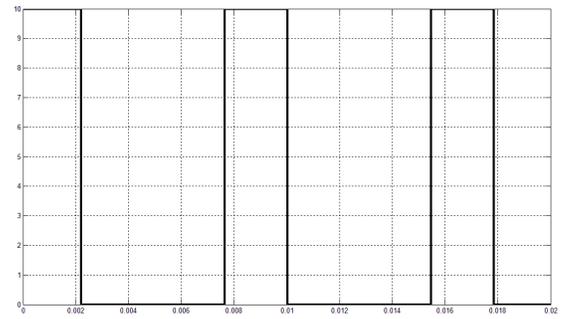
Figure 10 shows the waveforms of current and flux linkage in all phases and the gate pulse of one phase at 1000 rpm speed and 13.4118° gate pulse angle.

Table 5

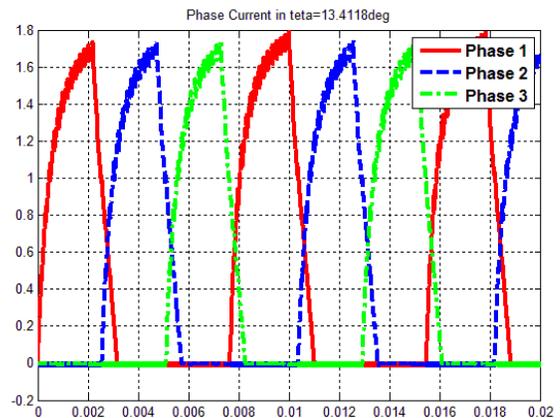
The best solutions of the proposed custom GA

	Value	$\theta$ (deg)	$T_{peak}$ (N.m)	$T_{bottom}$ (N.m)	$T_{ave}$ (N.m)
Best Torque (Maximum)	3.0145 N.m	13.4118	3.0145	0.8531	2.0897
Best TR (Minimum)	103.44%				

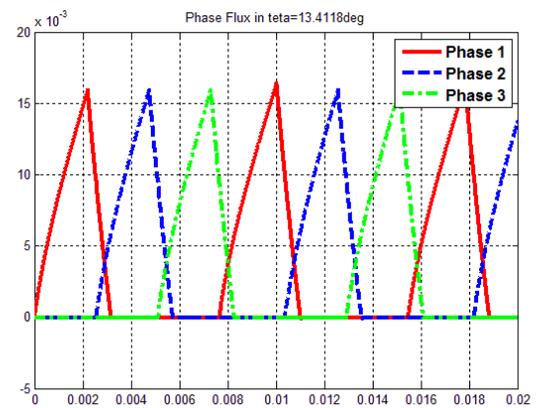
The generated torque profile of the selected SRM; in 1000 rpm speed and 13.4118° gate pulse angle, can be seen in Fig.11, which is much better than that obtained in 8° gate pulse angle.



(a)



(b)



(c)

Fig. 10. (a): Gate pulse of one phase, (b): Current and (c): Flux waveforms in the speed of 1000 rpm in  $\theta = 13.4118^\circ$

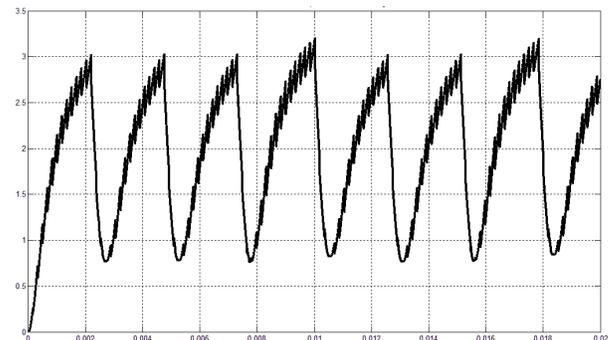


Fig. 11. Total generated torque of the SRM in the speed of 1000 rpm in  $\theta = 13.4118^\circ$

In the next stage, the SRM is simulated by using the proposed NSGA-II in the same conditions. Table 6 shows the best, worst and average values of torque which are achieved from the last population of the presented NSGA-II. Note that, these results are achieved after 19 iterations. Furthermore, the calculated optimization time is 290.6435s which are both lower than that was in the custom GA method. The maximum amount of torque is 3.2345 N.m which is achieved when the gate pulse of the SRM is 14.6471°. In addition, the minimum amount is 2.3875 N.m obtained in the gate pulse angle of 5.6471°. Moreover, the average torque is 3.1285 N.m

Table 6

The best, worst and average values of torque in the last population of NSGA-II

Best Toque		Worst Toque		Average Toque
Value (N.m)	$\theta$ (deg)	Value (N.m)	$\theta$ (deg)	Value (N.m)
3.2345	14.6471	2.3875	5.647	3.1285

The best and the worst values of TR in the last generation are presented in Table 7. The maximum value of TR is 192.76% which is obtained when the gate pulse angle is 11.8235. Furthermore, when the angle is 14.0588°, the minimum value of TR is obtained for the SRM which is 84.69%.

Table 7

The best and worst values of TR in the last population of NSGA-II

	TR Value (%)	$\theta$ (deg)	$T_{peak}$ (N.m)	$T_{bottom}$ (N.m)	$T_{ave}$ (N.m)
Worst TR	192.76	11.82	3.1044	0.0573	1.5808
Best TR	84.69	14.059	3.2343	1.3883	2.1799

The best solution is the gate pulse angle which forces the SRM to generate the highest value of torque and the least amount of TR. As is seen in Table 8, the best value for torque is achieved in 14.6471° angle; although, the best value of TR is in 14.0588° angle. These two angles are the best solutions which are shown in Table 9

Table 8

The best values of torque and TR achieved by the proposed NSGA-II.

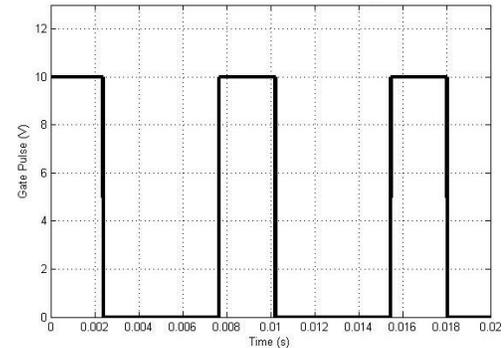
	Value	$\theta$ (deg)	$T_{peak}$ (N.m)	$T_{bottom}$ (N.m)	$T_{ave}$ (N.m)
Best Torque (Maximum)	3.2345 N.m	14.647	3.2345	1.388	2.1792
Best TR (Minimum)	84.69 %	14.059	3.2343	1.388	2.1799

Figure 12 illustrates the current and flux waveforms of all phases and the gate pulse of one phase in the speed of 1000 rpm and angle of 14.0588°.

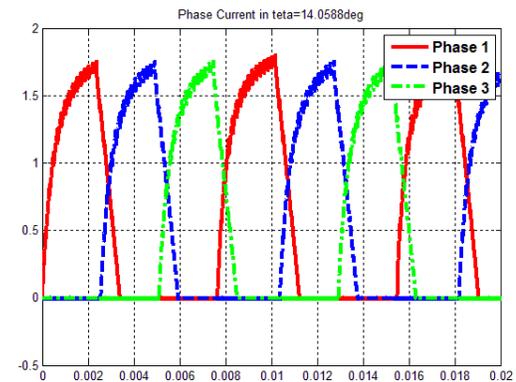
Table 9

The best solutions of the proposed NSGA-II

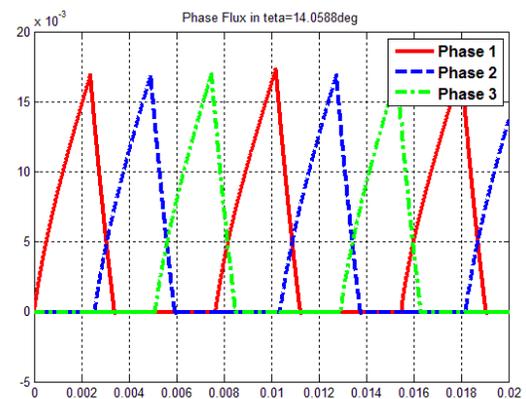
Best Solutions $\theta$ (deg)	TR (%)	$T_{max}$ (N.m)
14.6471	84.74 %	3.2345
14.0588	84.69 %	3.2343



(a)



(b)



(c)

Fig. 12. (a): Gate pulse of one phase, (b): Current and (c): Flux waveforms in the speed of 1000 rpm in  $\theta = 14.0588^\circ$

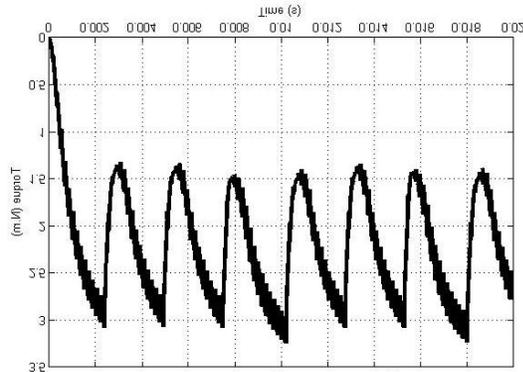


Fig. 13. Total generated torque of the SRM in the speed of 1000 rpm in  $\theta = 14.0588^\circ$

## VI. COMPARISON OF THE OBTAINED RESULTS

Table 10 summarizes the comparison of the results of the conventional, GA, and NSGA-II methods which are achieved in no-load condition and 1000 rpm speed of the selected SRM. As is shown, the optimization process time in NSGA-II is around 291s and the process is finalized in 19 iterations. However, GA performs the task in about 367s in 51 iterations. As is seen, the TR value which is generated in the proposed NSGA-II is approximately 210% less than that in the conventional method. Furthermore, the amount of the generated torque in the algorithm is roughly 1.13 times as great as that in the conventional strategy. By using the GA method, the generated TR is about 103% which is around 191% less than that in the conventional method. The maximum generated torque in GA is about 3.0145 N.m which is 1.06 times as great as that in the conventional method.

Table 10: The comparison of the final results of NSGA-II, GA, and conventional methods

	Irritation	Optimization Time (s)	$\theta$ (deg)	TR Value (%)	$T_{\text{peak}}$ (N.m)
Conventional	----	----	8	294.00	2.8586
GA	51	364.6791	13.41	103.44	3.0145
NSGA-II	19	290.6435	14.65	84.74	3.2345
			14.06	84.69	3.2343

## VII. COMPARISON OF THE PROPOSED NSGA-II METHOD WITH OTHER TECHNIQUES

In this section, the results of the presented NSGA-II method are compared with the three basic types of TR

reduction strategies which were explained in Section I. In comparison with “Changing the motor structure” techniques such as the ideas proposed in [4], [5], and [6], the presented method is very economical. This is because, in such methods, the mechanical structures of different parts of the motor such as the dimensions of poles, arcs, and yokes have to be changed after the first design process of the motor which is very hard and expensive.

A low-ripple torque control “Software” method is proposed in [12] which uses the TSF technique. In the study, different types of TSF configurations such as linear TSF, cubic TSF, exponential TSF, and online TSF are simulated and examined to decrease TR which results in interesting outputs. As a case in point, in the sampling time of  $5\mu\text{s}$ , the amount of the generated TR of linear TSF, cubic TSF, exponential TSF, and online TSF are respectively 100%, 93%, 80%, and 40%. In the presented NSGA-II method, the best value of TR is obtained around 84% which means it can be compared with the values resulted from TSF methods. However, TSF techniques require powerful processors; hence, they are usually very expensive in comparison with the presented idea of this paper.

In comparison with the “Changing the converter structure” method, the presented method is very economical again. For instance, the configuration proposed in [13] (in which a four-level converter is used to generate torque) uses a DSP chip for the calculations and gate pulse generation. Moreover, the four-level converter has a complex structure and needs more power switches in comparison with conventional converters. These highly increase the total cost of the driver circuit. On the other hand, in the presented idea, after obtaining the convenient fire angles for each range of speed, they can be placed in a look up table in the memory of a low-cost microcontroller to drive the motor by using a low-cost conventional converter and a neural network (or other similar) algorithm.

In summary, the presented NSGA-II method has the following merits in comparison with the other TR reduction and torque profile correction techniques:

- Fewer totals cost in comparison with other types of methods.
- Less computational cost in comparison with other “Software” techniques.
- Less complication in comparison with other types of methods.
- Higher performance speed in comparison with most of the other “Software” techniques and custom GA methods.

## VIII. CONCLUSION

TR is an inherent problem of SRMs that significantly decreases their popularity. Gate pulse angle is one of the important parameters whose value dramatically influences the quality of the generated torque profile of SRM. In this paper, the value of the gate pulse angle was optimized by NSGA-II to enhance the generated torque profile of a three-phase 12 by 8 SRM. The proposed control algorithm detected the best gate pulse angle based on the value of the generated torque and TR

of the motor for a desired speed of the motor. The results of the proposed idea were compared with the outputs of the conventional strategy of driving SRM, the custom GA method, and some other techniques proposed in other literature. By using the presented NSGA-II method, the value of the total generated torque of the machine was increased by around 13% in comparison with the conventional method of driving SRM. Additionally, the generated TR of the selected SRM was reduced by about 210% from 294% to 84.69%. It was shown that, in comparison with the other torque profile correction techniques, the presented method had the advantages of less computational and total cost, less complication, higher speed, and performance

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