A Trajectory Control Strategy for any Number of UAVs in Passive Localization of Radio Sources

Seyyed Ali Asghar Shahidian, Hadi Soltanizadeh

Abstract—In this paper an optimal trajectory control strategy is presented for single and multiple unmanned aerial vehicles (UAVs) equipped with received signal strength (RSS) sensors to localize a stationary RF source. The RSS at each UAV is observed in specified time instances. Due to the additive Gaussian noise caused by the non-line-of-sight (NLOS) propagation condition the location of the source is estimated using the extended Kalman filter (EKF). The objective is to determine the waypoints of the UAVs that minimize the source location uncertainty. The determinant of the Fisher information matrix (FIM) which is inversely proportional to estimation variance is applied to generate UAVs’ trajectories. The FIM is approximated at successive waypoints according to the estimated source location. To compensate the lack of adequate number of sensors when applying one or two UAVs the previous information is included in FIM calculations. The effectiveness of the proposed approach is depicted in simulation examples.

Index Terms—Received signal strength indicator, Fisher information matrix, Non-line of sight propagation condition, Extended Kalman filter.

I. INTRODUCTION

Aerial localization of radio emitters is an appropriate alternative for a variety of applications [1,2]. Since the radio signal emitters such as mobile phones are very common and most of the people carry one, this type of signal seems appropriate for the purpose of search and rescue missions [3-4]. The aerial localization is highly recommended to find a person who has been lost or needs help in avalanches, mountains, forests, etc. which are difficult to reach overland and have generally indirect vision.

Localization of an RF source could be classified based on the measurement approaches. Received signal strength indication (RSSI) [5], differential received signal strength indication (DRSSI) [6,7], angle of arrival (AOA) [8,9], time of arrival (TOA) [10-12], and time difference of arrival (TDOA) [13,14], are different parameters for passive radio source localization. RSS based localization is preferred in many applications including localization in sensor networks [15-17] and location based services in cellular phone networks [18,19]. Simplicity in hardware and implementation and computation of RSSI approach in RF source localization makes it preferable in variety of applications.

In addition to choosing the appropriate method and precise sensors and putting the sensors in locations with a direct vision of the signal, the localization geometry plays an important role to improve the localization accuracy. Maximizing the localization accuracy through optimizing the localization geometry is deeply investigated in literature [20-25]. The studies carried out in this area analyze the impact of the relative location of the sensors and the sources on the localization accuracy. Investigation of the optimal geometry in aerial localization with UAVs defines the trajectory control problem. The determinant of the FIM has been consistently applied as an observer control objective function in these studies.

The FIM, which is inversely proportional to the Cramer-Rao Lower Bound (CRLB), has been consistently used as an approximate optimization criterion for trajectory control of UAVs [20-23] and optimal localization geometry determination in Wireless Sensor Networks (WSNs) [24,25]. In the studies carried out around these topics the number of sensors/UAVs satisfies the minimum number that is required to enable the estimation of the source location in each waypoint individually. Three sensors are required to localize a radio source with RSS sensors in a single measurement instance.

In [20], a multi-UAV path planning approach for RSS based localization in NLOS condition is proposed to generalize the number of UAVs and develop the NLOS model of propagation. In [21] a real-time optimal UAV path planning is proposed for localization of multiple stationary RF emitters. In this research individual UAVs are assigned to teams that each team has adequate number of UAVs and is responsible to localize an individual source. A gradient based steering algorithm is adopted where the objective is to maximize the determinant of the approximated FIM using available emitter location estimates. In [22] a comprehensive study is performed on planning optimal paths for adequate number of UAVs localizing a single RF source with the purpose of maximizing the localization accuracy using the determinant of the FIM as the optimality criterion. In [21,22] a heterogeneous mix of sensors are applied.
A similar issue has been considered in wireless sensor network applications. In this type of studies the relative receiver-transmitter geometry and the effect of this geometry on the potential localization performance is investigated. A deep analysis of the optimal sensor-target geometries for range-only, time-of-arrival and bearing-only based localization has been performed in [24]. In [25] the optimality of the relative sensor-emitter geometry for signal strength based localization has been explored.

There are a lot of researches about optimality control of the trajectories of the UAVs that in all of them the adequate number of sensors are applied. To the best of the authors’ knowledge planning optimal path for the UAVs less than the adequate number has not been discussed in RF localization with the purpose of increasing the localization accuracy with RSS sensors. In this paper, a trajectory control strategy is proposed for any number of UAVs with RSS payload sensors trying to localize an RF source. The EKF is employed to estimate the location of the stationary source. To generate optimal UAVs’ waypoints direct maximization of the determinant of the FIM is applied. The FIM is approximated using current estimation of source’s location. Geometric path and movement constraints including the UAVs’ maximum turn rate and the maximum and minimum distance between UAVs are defined by inequality constraints.

This paper assumes that an RF source located in an unknown area needs to be located. One or multiple UAVs equipped with RSS sensors are appointed to localize the source with appropriate accuracy and speed. In search and rescue missions speed and accuracy are two important requirements. Accordingly, the key objective of this paper is to propose an autonomous control of the UAVs’ trajectories which help to maximize the accuracy and speed of the localization.

This paper is organized as follows. Section II provides a review of RSS based localization approach and the channel characteristics. Section III is dedicated to the EKF design for RF source localization using RSS observations. The proposed path planning approach for \( N \) UAVs is described in section IV in the presence of geometric path and movement constraints. In this section the localization uncertainty is described and the FIM is presented. Section V presents the simulation examples for the proposed UAV steering algorithm. Conclusions are drawn in section VI.

II. REVIEW OF RSSI APPROACH

In the free space line of sight channel with no obstacles nearby the transmitted signal attenuates since the energy is spread spherically around the transmitting antenna. Therefore the free-space path loss is proportional to the distance between the transmitter and the receiver of the radio signal [26],

\[
PL = PL(d_0) + 10\lambda \log\left(\frac{d}{d_0}\right),
\]

where \( d \) is the distance between the transmitter and the receiver, \( \lambda \) is the path loss exponent of the environment, \( PL(d_0) \) is the mean path loss in dB at distance \( d_0 \), and \( PL \) is the path loss in dB at distance \( d \). The relation between the transmitted signal strength \( (Pt) \) and the received signal strength \( (Pr) \) in dB is given by:

\[
Pr = Pt - PL.
\]

In real situations there may be an obstacle between the transmitter and the receiver. In this case which is called shadowing phenomenon, some part of the transmitted signal is lost through absorption, reflection, scattering, and diffraction. Therefore, the received signal is cumulative of reflected signals from different paths. Depending upon the phase of each reflected signal, these multiple signals may increase or decrease received power. The three components of the channel response, path loss, shadowing and multipath are shown in Fig. 1. To evaluate the received power correctly in NLOS condition, it is important to use appropriate models for multipath and shadowing.

---

**Fig. 1:** Signal attenuation due to multipath and shadowing
Multipath is referred to multiple received signals from multiple directions reflected from nearby obstacles. The effect of multipath on the received signal strength is nearly suppressed by averaging over the consecutive samples. Signal strength attenuation due to shadowing caused by large obstacles, such as hills, buildings or trees, has a normal distribution when it is expressed in dB. Therefore, the RSS observations are perturbed by zero mean white Gaussian noise (η) with standard deviation of \( \sigma_{db} \). The relation between the observed RSS (\( \overline{Pr} \)) and the true RSS (\( Pr \)) in dB is given by:

\[
\overline{Pr} = Pr + \eta.
\]  

III. SOURCE LOCATION ESTIMATION

Let \( s = [s^x, s^y]^T \) be the location of a stationary radio source in two-dimensional Cartesian coordinates. Superscript \(^T\) denotes the matrix transpose operator. The source localization uses RSS measurements collected by \( N \geq 1 \) UAVs at discrete time instants \( k \in \{0,1,...\} \). The position of the UAV \( j \) at time step \( k \) is denoted by \( u_k(j) = [u_k^x(j), u_k^y(j)]^T, j \in \{1,...\} \). The RSS at UAV \( j \) in dB at time step \( k \) (\( Pr_k(j) \)) is given by (4) in which \( Pt_k \) indicates the transmit power in dB,

\[
Pr_k(j) = Pt_k - PL_k(j).
\]  

The path loss (in dB) between the source and the UAV is given by [27]:

\[
PL_k(j) = PL(d_0) + 10 \log \left( \frac{||d_k(1)||}{d_0} \right),
\]  

where \( d_k(j) = s - u_k(j) \) is the range vector of the source from UAV \( j \) at time step \( k \) and \( ||\cdot|| \) denotes the Euclidean norm. For RSS based source localization the measurement equation is:

\[
\overline{Pr}_k = Pr_k(s) + w_k,
\]  

where \( \overline{Pr}_k \) is the vector of RSS measurements taken by \( N \) platforms at time step \( k \), \( w_k \) is white Gaussian noise \( w_k \sim N(0,R_k) \), and \( Pr_k(s) \) is the true RSSs at time step \( k \):

\[
H_k = \begin{bmatrix}
\frac{\partial Pr_k^1}{\partial s^x} & \frac{\partial Pr_k^1}{\partial s^y} \\
\vdots & \vdots \\
\frac{\partial Pr_k^N}{\partial s^x} & \frac{\partial Pr_k^N}{\partial s^y}
\end{bmatrix}_{s^x = s_{k-1}^x, s^y = s_{k-1}^y}
\]  

The EKF recursions are initialized by \( s_0 = E[s] \) and \( P_0 = \text{cov}[s] \)

IV. UAV TRAJECTORY CONTROL

A. Problem Description

It is considered that an RF source located at \( s = [s^x,s^y]^T \) is going to be localized by \( N \geq 1 \) UAVs equipped with RSS sensors. It is assumed that the UAVs and the source are coplanar so a two-dimensional model are applied. The UAVs can communicate with each other and the calculations are performed in a processing unit, which may be located in one of the UAVs. The locations of the UAVs are known by the use of the Global Positioning System (GPS) in each moment. The UAVs attempt to measure the RSSs at known time steps \( k = 0,1,... \) and the processing unit estimates the location of the source.

At each waypoint the UAVs observe the RSSs subjected to the source. According to the new location estimate the processing unit determines the next movement of the UAVs.
at discrete time instants. According to the FIM criterion, the UAVs move to the locations which minimize the error variance of the source location estimation. The proposed UAV waypoint update algorithm is as follows:

\[ u_{k+1}(j) = u_k(j) + v_k(j), \quad i = 1, \ldots, N \]  

where the UAV waypoint update \( v_k(j) \) satisfies the speed and steering constraints as (12). \( T \) is the time interval between each consecutive RSS sampling and waypoint update,

\[ r_{\text{min}} \leq \|u_k(j) - u_k(i)\| \leq r_{\text{max}}, \quad i \neq j, \]

(13)

\[ \|v_k(j)\| = vT, \]

\[ |\angle v_{k+1}(j) - \angle v_k(j)| \leq \varphi, \]

(12a)

(12b)

where \( \varphi \) shows the maximum turn rate of the UAVs at speed \( v \). Without loss of generality the speed of the UAVs are considered equivalent to each other and constant in all instants. In addition to turn rate and speed limitations, two hard constraints which prevent collision and assures the connectivity between UAVs are defined as (13). Fig. 2 illustrates the proposed UAV waypoint update subject to the geometric path constraints.

Based on the RSS measurements at each time step, the location of the source would be estimated with the EKF. Crammer-Rao Lower Bound defines the lower bound on the estimation variance. The waypoint update of each UAV is performed based on the estimation uncertainty of the source’s location.

B. Localization Uncertainty

The CRLB states that the inverse of the Fisher Information is the lower bound of the error variance of any unbiased estimator. Since the probability density function has Gaussian distribution, the Fisher information would be as:

\[ I(U_k) = I(U_{k-1}) + \hat{B}_k^2(R_k)^{-1}\hat{B}_k, \quad U_k = \begin{bmatrix} u_k(1) \\ \vdots \\ u_k(N) \end{bmatrix} \]

(14)

where \( \hat{B}_k \) is the Jacobian matrix of the measurements and \( R_k \) is the RSS measurements noise covariance matrix (8). In each time step the FIM is approximated using the source location estimate. The Jacobian at time step \( k \) for an RF source would be as:

\[ \hat{A}_k = -\frac{10\lambda}{\ln(10)} \begin{bmatrix} \|\hat{d}_k(1)\|^2 & s_k^x - u_k^x(1) \\ \vdots & \vdots \\ \|\hat{d}_k(N)\|^2 & s_k^x - u_k^x(N) \end{bmatrix} \]

(15)

where \( \hat{d}_k(j) = s_k - u_k(j) \). The determinant of the FIM is used as an objective function for the proposed path planning approach.

To determine the next waypoint of the UAVs, the FIM should be maximized over the next UAVs’ locations. The possible waypoint update for each UAV would be expressed as:
\[
\mathbf{v}_k(i) = v_T \begin{bmatrix} \cos(\theta_k(i)) \\ \sin(\theta_k(i)) \end{bmatrix}, \quad i = 1, \ldots, N
\]  
(16a)

\[
\mathbf{V}_k = [\mathbf{v}_k(1), \ldots, \mathbf{v}_k(N)]^T,
\]  
(16b)

\[
|\theta_{k+1}(i) - \theta_k(i)| \leq \varphi,
\]  
(16c)

where \(\theta_k(i)\) is the angular direction for the control vector assigned to UAV \(i\) at time step \(k\).

The cost function \(|\mathbf{I}(\mathbf{U}_k)|\) gives a measure of the location estimation uncertainty. \(|\mathbf{I}(\mathbf{U}_k)|\) is a function of the UAV locations \(\mathbf{U}_k\), the measurements error covariance, and the state estimate \(\mathbf{s}_k\) at time step \(k\). The UAV waypoint update can be formulated as:

\[
\begin{bmatrix}
\hat{\theta}_k(1) \\
\vdots \\
\hat{\theta}_k(N)
\end{bmatrix} = \arg \max_{\theta_k(i)|\mathbf{u}_k} |\mathbf{I}(\mathbf{U}_k)|,
\]  
(17)

where \(\hat{\theta}_k(i)\) is the optimal angular direction for steering the UAV \(j\) to the next waypoint. The selected waypoints maximize the determinant of the Fisher information. The proposed algorithm in (17) requires the maximization of \(|\mathbf{I}(\mathbf{U}_k)|\) over \(\theta_k(i)\) at each waypoint update. Regardless of the maximization approach, an appropriate initialization strategy should be applied to avoid convergence to a local maximum. An alternative is to propel the UAVs to the source, i.e. move the UAVs towards the centroid of the known search area.

V. SIMULATIONS

The proposed approach has been simulated in NLOS condition with \(N \geq 1\) UAVs. The RF source is located at random location in a circular area with 7 Km of radius. The initial location estimation of the source is considered at random location in the search area. The initial localization uncertainty, i.e. the covariance of estimation in the EKF is set in such a way that the entire search area is covered. The UAVs search the area with the speed of 150 km/h and attempt to form the DRSS measurements every 10 seconds, \(T = 10s\). Estimation of the location of the source is updated based on the RSS measurements. Afterwards, the proposed method determines the next waypoint of each UAV. The standard deviation of shadowing which disturbs the RSS measurements is assumed to be 3dB. We set \(d_0 = 10m\), \(PL(d_0) = 12.1\,db\), and \(\lambda = 3\). The maximum turn rate for the UAVs is \(\varphi = 40^\circ\) (i.e. \(\frac{40^\circ}{\sec}\)) and the RMSE for emitter location estimates at time step \(k\) is given by:

\[
RMSE_k = \sqrt{\text{trace}(\mathbf{P}_k)}
\]  
(18)

where \(\mathbf{P}_k\) is the filtered estimation covariance of the Kalman filter at time step \(k\). The initial locations of the UAVs at the beginning of the localization mission in supposed virtual coordinate system are:

\[
\mathbf{u}_k(1) = [2, 3]^T \, \text{km},
\]

\[
\mathbf{u}_k(2) = [1, 4]^T \, \text{km},
\]

\[
\mathbf{u}_k(3) = [1, 2]^T \, \text{km}.
\]  
(19)

The resulted path for different number of UAVs is illustrated in Fig. 3(a-c) and the evolution of the RMSE for each case is depicted in Fig. 3(d).
Figure 3 shows the trajectories of different number of UAVs in localization of an RF source. In the proposed approach the UAVs initially are not on the path toward the source, rather they move away from each other for better localization geometry. After more than half-way through the UAVs gradually reduce their distance to each other and approach the estimated location of the source and eventually reach the source. Fig. 3(d) shows the evolution of the RMSE at each time step.

VI. CONCLUSIONS

In this paper a UAV trajectory control approach has been developed for RF source localization using $N \geq 1$ UAVs equipped with RSS sensors. The RSS measurements were applied as an input for the developed EKF. The proposed steering algorithm determines the next waypoints of the UAVs based on the FIM criterion. The determinant of the FIM specifies the localization accuracy relative to the sensor target geometry. The proposed trajectory control autonomously defines the waypoints of the UAVs in order to improve the localization geometry. The optimal trajectory control prevents the UAVs from the geometries with low estimation accuracy. The proposed approach enables controlling the trajectories of any number of UAVs.

REFERENCES


