A Novel Space Reduction Technique for Design Optimization of Permanent Magnet Synchronous Motors

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Abstract— **Due to the non-linearity and large dimensions of permanent magnet motor optimization, the use of metaheuristic methods such as GA, PSO, etc. would not be the most appropriate method especially if the fitness assessment is done by a time-consuming solver such as finite element analysis (FEA). The FEA, which is widely used by most researchers, requires a lot of time and space and leads to huge computational costs. On the other hand, the accuracy of approximate analytical models is not sufficient for high-dimensional optimization tasks. To overcome these problems, a new space reduction optimization method is developed and presented in this paper. The proposed method gradually shrinks the search space and approaches an interesting subspace so that the wide variable space becomes smaller. As a result, FEA modeling accuracy is achieved as well as computational cost reductions. To validate the method, the design optimization is performed on a 2004 Toyota Prius Interior Permanent Magnet (IPM) motor. The results are compared with other optimization algorithms in terms of accuracy and number of performance evaluations. The comparison results show the superiority of the proposed algorithm, which can be a desirable alternative to industrial optimization tasks that necessarily require the least number of function evaluations.**

Index Terms— ***Design Optimization, Space Reduction Technique, Problem Dependent Optimization (PDO), Finite Element Analysis (FEA), Interior Permanent Magnet (IPM).***

I. INTRODUCTION

Design optimization of industrial appliances is divided into two categories based on the type of model solution. In the first part, analytical models are used to solve the problem, which may not be the best representation. Most optimization tasks in these cases are performed with meta-heuristic algorithms. These analytical models are less accurate and are not supported for electromagnetic devices due to nonlinear behavior. The second part of the research uses numerical modeling such as finite element analysis (FEA) which is time-consuming. Therefore, meta-heuristics will not be a good choice. Instead, heuristic methods (problem-dependent optimization) are better benchmarks for such applications, especially for optimizing FEA paired design.

The design of electromagnetic devices is an industrial engineering subject in which both meta-heuristic and heuristic methods are proposed and studied for various industrial applications. The optimal design of permanent magnet (PM) machines is one of the controversial topics in electromagnetic design in which the need for efficient optimization algorithms is necessarily felt [1] - [2].

Based on the above discussion, the first part of the study uses meta-heuristic optimization algorithms to optimize the permanent magnet motor in which the problem is solved with analytical models. For example, Ref [3] used a hybrid multi-objective design optimization for a permanent magnet motor, which is an optimization algorithm based on the artificial bee colony technique (ABC), Strength Pareto Evolutionary Algorithm (SPEA2), and Differential Evolution Strategy (DE). In another study [4], a multi-objective technique was performed on an interior PM motor in which the efficiency, volume, and cost of the motor were considered as a fitness function. Then the best solutions are obtained as Pareto-front and the k-means clustering algorithm is used to extract the final solutions. In Ref [5], the design optimization of an interior v type PMSM is presented to find all local and global optimizations. The objective functions of the research are to minimize the total weight and losses of the motor and mechanical, thermal, and magnetic constraints. In another study [6], the design and validation of a PM motor were performed through an analytical method based on the conformal mapping technique. In Ref [7], the multi-objective design optimization is performed by the Non-dominated Sorting Genetic Algorithm-II (NSGA-II) for a surface PM motor, which aims to reduce the clamping torque and stabilize the output torque. Ref [8] presents a new genetic algorithm combined with a proposed sub-domain model to improve performance for surface PMSM to optimize magnetic field distribution, cost, and motor efficiency. A saturated surface-mounted PMSM is optimally designed through a multi-objective approach [9]. Different PMSM rotor structures are investigated using a new multi-objective optimization technique [10]. A hybrid differential evolution algorithm has been proposed for the optimal design of high-speed PMSM as an electric vehicle propulsion system [11]. The design optimization of a permanent magnet synchronous motor with hybrid permanent magnets has been proposed by considering

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irreversible demagnetization [12]. A new algorithm based on particle swarm optimization is proposed along with an adaptive direct search method to optimize the design of a permanent magnet synchronous motor [13].

Many of the above researches have encountered problems in modeling the nonlinear behavior of PM motors, including sub-domain model, conformal mapping, or a modified magnetic equivalent circuit (MEC) method. The FEA, on the other hand, can easily consider these nonlinearities, which is the subject of the second part of the research. This research relates to problem-dependent optimization (PDO) methods that have been customized for specific applications. In these problems, instead of analytical modeling, numerical modeling such as finite element analysis (FEA) is used. As a result, due to the higher accuracy of FEA modeling compared to analytical modeling, more accurate solutions will be produced. However, the FEA is time-consuming in limiting the number of function evaluations (NFEs) in the optimization process. Therefore, the main obstacle to using FEA as a solution to the design optimization task model is limited NFE. There are ways to overcome this deficiency in the literature. Multilevel optimization is one of the methods used to optimize the design of electromagnetic devices [14] - [15]. Two types of variables called upper-level variables and the lower-level variables are involved in these issues. The problem is then solved for each set of design variables, so NFE is effectively reduced. Another approach in this field is the use of space mapping methods to optimize the design of electromagnetic devices [16] - [18]. Space mapping involves a very fast coarse model with a time-consuming fine model. As a result, direct optimization and time on the good model are avoided. In other words, the optimization is done in a coarse space and then it is mapped into a fine space. Surrogate models are also one of the new optimization methods in the optimal design of electromagnetic devices [19] - [23]. In these methods, an approximate model is made based on sampling. This method mimics the behavior of a simulation model, which leads to faster performance evaluation. Sequential optimization methods are also used in optimizing the design of electromagnetic devices [24] - [27]. In these methods, the search space is gradually reduced to prevent performance evaluation in the unnecessary search space. Therefore, the NFE is significantly reduced which is suitable for problems with high computational costs.

The main difference between meta-heuristic and PDO methods is that PDOs are customized and developed to be used in a particular problem, while meta-heuristics cover a wide range of applications. This limits meta-heuristics performance and makes them less efficient in certain areas that require special consideration. Optimizing the design of electric machines with finite element analysis necessitates the development of a suitable and efficient optimization algorithm with the least number of performance evaluations.

This paper proposes a space reduction technique to be used in FEA-based design optimization. The proposed technique gradually shrinks the search space and approaches an interesting subspace in which it is optimally placed. This algorithm can be used to optimize the design of electromagnetic devices. To demonstrate the efficiency of the proposed algorithm, it has been used for an electromagnetic design task, namely the optimal design of the 2004 Toyota Prius IPM motor. The rest of the article is organized as follows.

In the second part, the proposed method of reducing space is discussed and explained. Section III presents the IPM motor optimization process while the optimization results are presented in Section IV. Finally, the article concludes in Section V.

II. Description of the Proposed Method

This section describes the proposed method. First of all, each optimization algorithm is evaluated based on three indicators:

- Accuracy: obtain a solution with precision
- Robustness: cover a wide range of problems in different fields
- Efficiency: require the least computer time or storage.

These features are often in contrast. For example, a robust method is slow because it covers a wide range of problems. There must be an engineering balance between convergence rate and storage needs, between strength and speed, etc. [28] - [30].

In the proposed algorithm, finite element analysis is used as the model solution, which is more accurate and the concept of space reduction is used to reduce NFE. In design optimization work, which is accompanied by finite element analysis, the most important indicator after accuracy is the efficiency of the algorithm. In these cases, the algorithms do not need to be robust, but we need to get the optimal response in the least computer time or several function evaluations (NFEs). Space reduction methods are among the problem-dependent optimization techniques that are suitable for optimizing the design of electric machines with FEA.

The search space in the design of electric cars is very large, which includes all possible solutions. However, the optimal solution lies in a small sub-space (interesting sub-space). The goal of space reduction methods is to find an interesting subspace and avoid evaluating performance outside of it. This in turn can drastically reduce the number of performance evaluations.

In the following sections, the process of the proposed algorithm is presented.

A. Basic Methodology

This algorithm aims to find the minimum of the function which is written in the following generic form:

$$
\begin{aligned}
&minimize\ f_i(x),\ x \in \mathbb{R} \quad (i = 1,2,\dots,M) \qquad (1)\\
&subject\ to\ h_j(x) = 0, \quad (j = 1,2,\dots,J)\\
&g_k(x) \le 0, \quad (k = 1,2,\dots,K)
\end{aligned}
$$

Where $f_i(x)$, $h_j(x)$ and $g_k(x)$ are functions of the problem input vector

$$x = (x_1, x_2, \dots, x_{N_{var}}) \qquad (2)$$

This method divides the search space into securely organized subspaces and evaluates the objective function in each. For a simple description of this method, we consider a bivariate problem. The search space is divided into N sections (N = 4 for a bivariate problem). In general, a permutation matrix is formed that includes all possible members. The fit function is then evaluated at the center of these N segments, and the section with the best fit is extracted (Fig. 1(a)). After that, the range of design variables is updated according to the following formula:

$$x_{min}{}^{t+1} = x_{opt}{}^{t} - ((\frac{k_{div}}{2}) \times stepx^{t}) \quad (3)$$

$$x_{max}{}^{t+1} = x_{opt}{}^{t} + ((\frac{k_{div}}{2}) \times stepx^{t}) \quad (4)$$

where k_{div} is the division factor of variables, x_{opt} the point corresponding to the best fitness, x_{min} the minimum of the variable range, x_{max} the maximum of the variable range. Besides, stepx is defined as:

$$stepx^{t} = \frac{(x_{max}{}^{t} - x_{min}{}^{t})}{k_{div}+1} \quad (5)$$

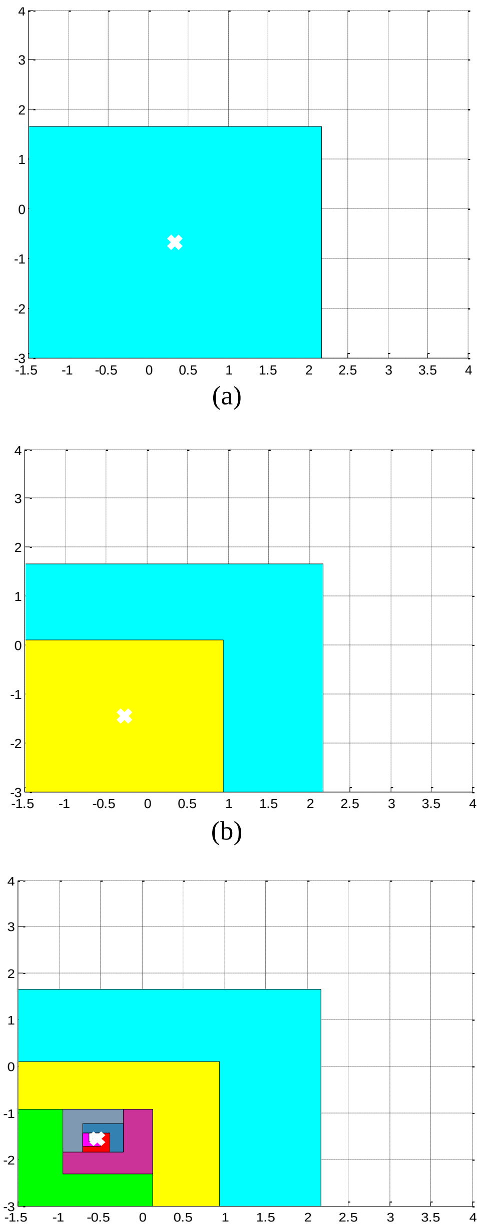

Fig. 1. The procedure of space reduction in the proposed Algorithm (a) Stage 1 (b) Stage 2 (c) Final Stage

As a result, depending on the value of k_{div}, the search space is reduced. This process is repeated until the optimal point is found. Fig. 1(b) shows the second step of the algorithm and the whole steps of the algorithm are shown in Fig. 1. (c). The method flow diagram is shown in Fig. 2.

Table I gives the results of the algorithm for some well-known optimization benchmarks. If each design variable is divided by a factor k_{div}, the NFE can be calculated as follow:

$$NFE = iter \times (k_{div}{}^{N_{var}}) \quad (6)$$

Where N_{var} is the number of variables and iter the stages of the algorithm. If the accuracy has to be enhanced, the factor k_{div} and iter should be increased which in turn increases the NFE and computational cost. Table II compares the influence of these two factors on the accuracy and NFE of a test function i.e. McCormick benchmark. As a result, it can be cited for complicated functions, it is appropriate to increase k_{div}. In complicated functions, there are multiple extremums and increasing k_{div} ensures that the algorithm covers most of the search space. The parameter iter is related to the accuracy of the solution obtained. In general, these parameters should be selected based on the required performance and accuracy.

TABLE I
Results of the Algorithm for Optimization Benchmarks

Benchmark	**Optimal Values by the proposed method**	**NFE**	**Desired value**
Sphere	F(0.001,0.001)=2e-6	52	F(0,0)=0
Booth	F(1.01,3.01)=0.003	52	F(1,3)=0
Matyas	F(-0.01,-0.01)=5.7e-6	44	F(0,0)=0
Himmelblau	F(2.999,2.007)=7.3e-4	56	F(3,2)=0
Three-hump camel	F(0.006,-0.006)=7.19e-5	44	F(0,0)=0
Ackley	F(0,0)=0.0001	52	F(0,0)=0
Goldstein-price	F(0.004,-1.01)=3.05	44	F(0,-1)=3

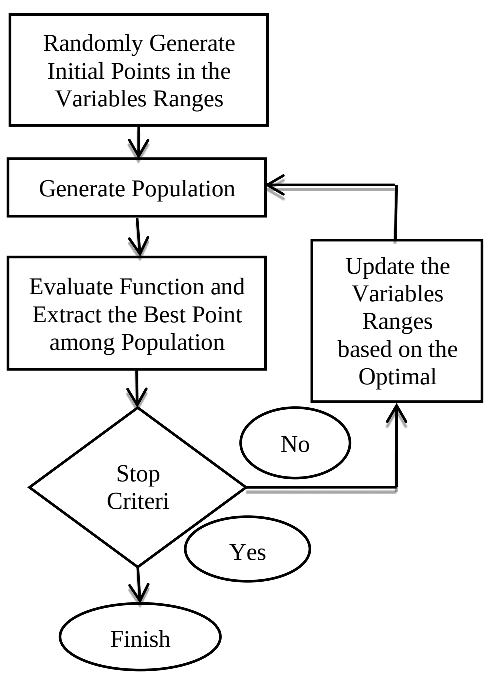

Fig. 2. Flowchart of the proposed Algorithm

TABLEII
Results of the Algorithm for Different K_{div} and iter in McCormick Benchmark

k_{div}	2	3	3	4
iter	10	6	7	5
NFE	40	54	63	80
Optima	-1.9130	-1.9121	-1.9131	-1.9132

B. Generalization of the method for high-variable problems

As equation (6) suggests, the NFE would rise drastically if the N_{var} is increased beyond a certain value. In practical applications such as the optimal design of IPM motor, NFE climbs up the 1000 evaluations for $N_{var}>6$. This amount of NFE is not feasible for FEA-based design optimizations. As a result, some modifications have to be made to make use of the method for high-variable problems. Two techniques are proposed in this algorithm as follows.

(2.1), Scheme-LHS

The first method is to reduce permutation matrix elements in such a way that their accuracy is not affected. To do this, a powerful sampling method is used, i.e. the Latin Hypercube (LHS) sampling. In this context, a certain number of possible compounds and not all of them are considered. Although this may lead to inaccurate results, LHS is used to ensure more search space coverage. The LHS provides a uniform distribution of data. As a result, NFE is reduced while accuracy is not significantly affected.

(2.2), Scheme-Fibonacci

The second technique is to reduce the number of evaluations per step of the algorithm based on the Fibonacci series. In this method, in the first stage, all possible elements of the permutation matrix are evaluated and in the next stages, the number of evaluations is reduced step by step. Part of the Fibonacci series is {1,1,2,3,5,5,8,13,21,34,55,…..} The declining trend is inspired by the series leading to the following formula for the number of populations at each stage of the optimization loop:

$$N_{pop} = fix\left\{\left(\frac{k_{div}{}^{N_{var}}}{100}\right) \times (145.4e^{(-0.4979\times iter)} + 0.9572)\right\} \quad (7)$$

Where the coefficients are obtained by curve fitting.

III. Optimization Process of the 2004 Prius IPM Motor using the Proposed Algorithm

The proposed algorithm is applied to design optimization of a specific electromagnetic device i.e. Prius IPM motor.

Fig. 3 shows the optimization workflow using the proposed algorithm. To pair, design optimization must be related to finite element analysis, MATLAB, and Ansys Maxwell must be relevant. The script is written in MATLAB and Ansys Maxwell solves the solutions. The solutions are then returned to optimization to generate the next stage population.

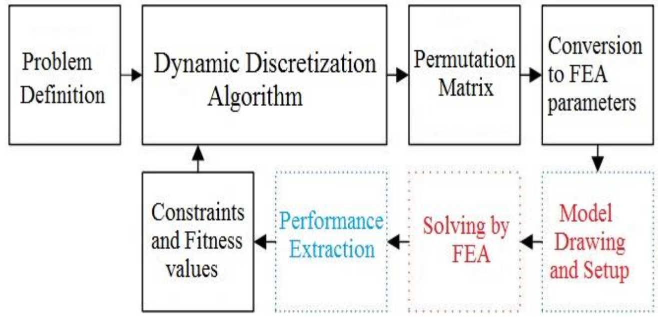

Fig. 3. Workflow of the optimization

(3.1), IPM motor description

Fig. 3 shows the topology of the IPM motor in which permanent V-shaped magnets are placed in the rotor. The specifications of the mentioned engine are given in Table III. As shown in Fig. 4, five design variables are selected. The maximum and minimum ranges of these five design variables are given in Table IV.

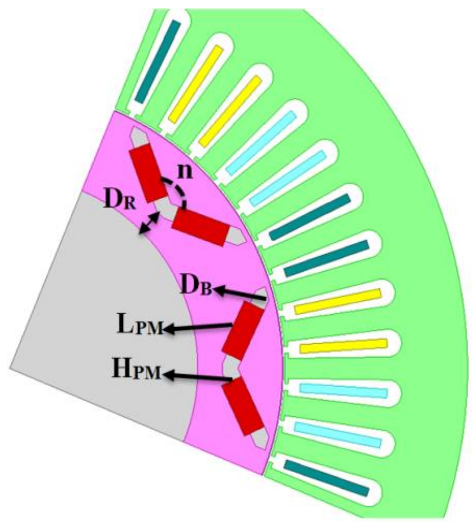

Fig. 4. The topology of the 2004 Prius IPM motor

TABLE III
Specification of the 2004 Prius IPM Motor

Parameter	**Value**
Stator outer/inner radius (mm)	134.6/80.95
Rotor outer/inner radius (mm)	80.2/55.32
Stack length (mm)	83.82
Rated phase current peak value (A)	250
Maximum current (A)	400
Rated speed (rpm)	1500
Rated torque (Nm)	305
Torque Pulsation (%)	26
Maximum torque (Nm)	400
B_r (T) at 50 _C	1.19
H_c (kA/m)	-920
Relative permeability (μ_r)	1.03

TABLE IV
Maximum and Minimum Ranges of the Design Variables

Parameter	Min	Max
L_{PM} (mm)	12	18
H_{PM} (mm)	5	8
D_R (mm)	6	8
D_B (mm)	3	5
n/2 (deg)	55	80
Stator outer radius (mm)	120	160
Stator inner radius (mm)	70	90
Stack length (mm)	70	95

The objective function of the optimization is to minimize the permanent magnet volume

$$F = min\,\{V_{pm}\} \tag{8}$$

Where the average torque and torque ripple constraints have to be met

$$T_{pulsation} \le T_{pulsation,base}\,,\ T_{avg} \ge T_{avg,base} \tag{9}$$

(3.2), Finite element modeling

The step-by-step finite element method is the most accurate tool available for analyzing electric machines. In this paper, Ansys Maxwell is used as the FE solvent. To reduce the simulation time, an exchange is made between the computation time and the reliability of the results. As a result, an average density of meshes and an average time step are chosen for the simulation. Another factor that affects the simulation results is the magnetic saturation of the ferromagnetic core. The simulation time for each run is about 60 seconds. The simulation results are presented in the next section along with the optimal structure.

IV. Optimization Results and Discussion

In this section, optimization results are obtained for two scenarios. In the first scenario, a 5-variable optimization problem is considered and for the second scenario, an 8-variable problem is considered. The first scenario is optimized with the initially proposed method and the second scenario is applied for Scheme-LHS and Scheme-Fibonacci. Both scenarios are compared and approved by GA.

(4.1), Scenario 1

Optimization is performed for the first 5 variables of Table IV. The next three variables are fixed. The volume of permanent motor magnets in 139095 mm3 is obtained after 480 function evaluations with acceptable accuracy. Relative tolerance of response 0.01 was selected. The trend of the value of the objective function in front of each step of the algorithm, i.e. iteration is shown in Fig. 5. The optimal dimensions and performance characteristics of the IPM motor are obtained and then the proposed algorithm is compared with GA. The comparison results are given in Table V. It is shown that both algorithms converge to the same optimal response. However, the proposed method required only 480 NFE while GA required 2500 NFE to achieve the optimal response.

Fig. 6 and 7 show the optimal motor flux line and flux density in 2D FEA software. Fig. 8 shows the flux connection of a motor phase. The output torque, which consists of electromagnetic torque and reluctance torque, is shown in Fig 9.

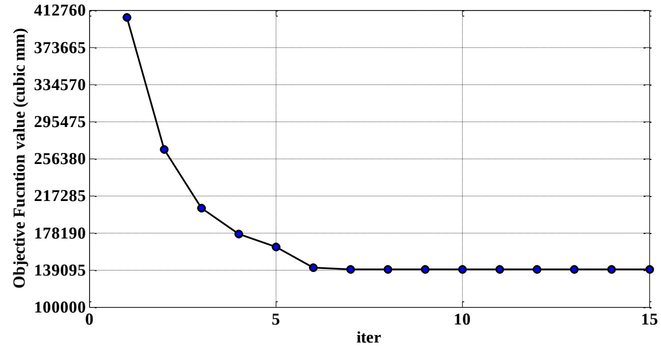

Fig. 5. The trend of objective function versus iteration

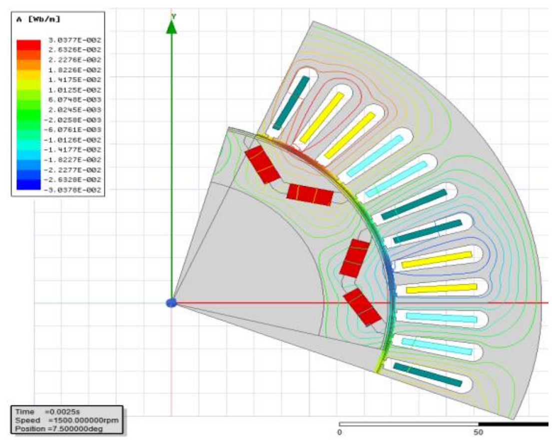

Fig. 6. Flux lines diagram of the optimal motor

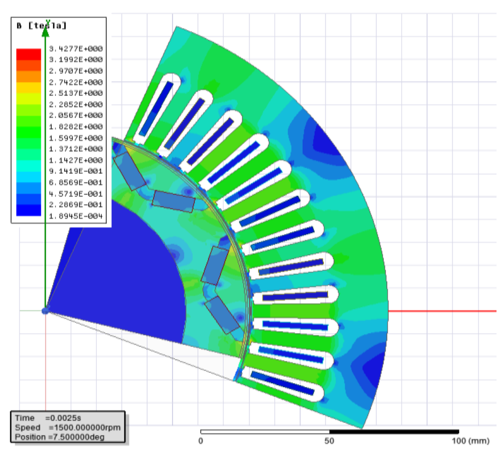

Fig. 7. Flux density diagram of the optimal motor

(4.2), Scenario 2

In this scenario, all 8 variables in Table IV are considered for optimization. Due to a large number of variables, Scheme-LHS and Scheme-Fibonacci are used and then compared with the original proposed method and GA. According to the relative tolerance of optimization, Scheme-LHS and Scheme-Fibonacci require 750 and 593 estimates, respectively (Table VI). This evaluation value is 2500 for the proposed base method and 8000 for the GA. The obtained results show the acceptable accuracy of the proposed method for high-variable optimization problems.

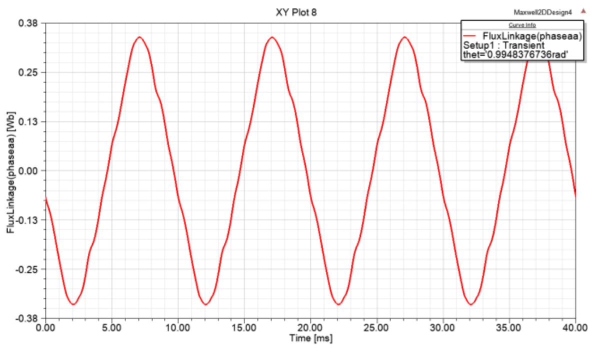

Fig. 8. Flux linkage waveform of the optimal motor

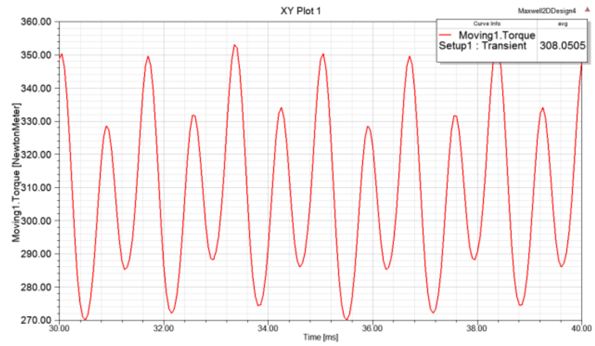

Fig. 9. The output torque of the optimal motor

TABLE V
Comparison of the Proposed Method and GA for Scenario 1

Parameter\Method	GA	Proposed method
Rated torque (Nm)	308	307
Torque Pulsation (%)	25	25.5
L_{PM} (mm)	17.2	17.15
H_{PM} (mm)	6.03	6.04
D_R (mm)	7.45	7.44
D_B (mm)	3.9	3.92
n/2 (deg)	68.4	68.5
NFE	2500	480
CPU time (h)	40	8

TABLE VI
Comparison of the Scheme-LHS, Scheme-Fibonacci, Basic Proposed Method and GA for Scenario 2

Parameter\Method	Scheme-LHS	Scheme-Fibonacci	Proposed method	GA
Rated torque (Nm)	308.2	307.5	308	308
Torque Pulsation (%)	25.1	25.4	25	25
L_{PM} (mm)	17.2	17.1	17.3	17.2
H_{PM} (mm)	6.03	6.05	6.02	6.03
D_R (mm)	7.45	7.35	7.4	7.41
D_B (mm)	3.9	4	3.95	3.93
n/2 (deg)	68.4	69	68.2	69.1
Stator outer radius (mm)	135	134.5	135.1	134.8
Stator inner radius (mm)	81.5	80.7	81.2	81.3
Stack length (mm)	83.9	84.4	84.1	83.8
NFE	750	593	2500	8000
Time (h)	12.5	10	40	133

Computer used: Intel ® CPU (2.2 GHz, 5 cores) and 8 GB RAM

V. Conclusion

In this paper, a new optimization method based on the space reduction technique is proposed to optimize the design of a solved PMSM with finite element analysis. The proposed algorithm gradually reduced the search space and approaches to an interesting subspace so that many non-executable variable combinations were rejected. This method was successfully tested in the Prius 2004 IPM motor and compared with other optimization algorithms such as GA in terms of accuracy and number of performance evaluations. The following results are obtained:

• Successful implementation of the proposed algorithm for optimizing IPM motor design with FEA to gain higher accuracy.

• Superiority of the proposed algorithm compared with well-known meta-heuristic algorithms such as GA in terms of optimization algorithm efficiency

• Generalization of the proposed algorithm for high-variable optimization problems which was verified by two practical techniques

• Generality of the proposed method for other machine types especially industrial optimization tasks which require the least number of function evaluations.

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