Designing a Robust Memory State Feedback Controller Leveraging LMI on DC Motor Under Time Delays and Norm-Bounded Disturbances

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Abstract*-- This paper investigates the performance of a predictor-based H_∞controller applied to a first-order DC motor system under significant input-time delays. Alongside the proposed predictor-based H_∞ controller, an LQR controller has been implemented to serve as a benchmark for effectively comparing and evaluating the performance of the proposed method. This study specifically focuses on scenarios involving time delays exceeding one second and external disturbances such as constant, sinusoidal, and stochastic disruptions. The proposed controller employs a robust memory-state feedback mechanism to ensure closed-loop stability and minimize the impact of disturbances. Using Linear Matrix Inequality (LMI) conditions, the controller compensates for input delays of up to five seconds while guaranteeing disturbance attenuation. A delay-dependent Lyapunov stability analysis is conducted to validate the proposed approach. Comprehensive simulation results and evaluations based on key performance metrics, such as settling time and overshoot, indicate that the predictor-based H_∞ controller outperforms both the open-loop configuration and the LQR controller. Notably, the proposed approach achieves a reduced overshoot, a faster transient response, and superior disturbance attenuation compared to the LQR method. Furthermore, this controller significantly enhances the overall system robustness and control precision in scenarios with prolonged delays. The predictor-based H_∞ controller suggests an innovative solution for mitigating the effects of long input delays in DC motor systems, thereby overcoming a pivotal challenge in robust control.

***Index Terms*- Robust Controller, Time delay system, Norm Bounded Disturbance, LMI, Convex Optimization, LQR**

I. Introduction

Time delays are a common challenge in control systems, often resulting from the interval needed to collect necessary data, formulate control commands, and execute these actions. Time delay is not limited to engineering applications; it is frequently encountered in various domains, such as biological, medical, chemical, and economic processes. Additionally, time delays are intrinsic to the modeling of physiological systems, ecological systems, population dynamics, and in fields like transportation and communication [1-3].

Time delays are frequently a source of instability, making the stability analysis and robust control of time-delay systems crucial from theoretical and practical perspectives [4-5]. The conventional approaches to stability analysis for time-delay systems are the Lyapunov-Krasovskii and Lyapunov-Razumikhin methods [6-8]. Although the Lyapunov-Razumikhin method is easier to implement, the Lyapunov-Krasovskii approach is less conservative, making it the preferred method in most recent studies and publications [9, 10]. Time delays can occur in the system states, in the control input, or both. However, the majority of previous studies have primarily considered delays within the system states [11, 12]. For systems with delays in the control input, the actuator introduces a delay before transmitting the control signal directly to the system [13]. Utilizing predictor-based feedback is an effective approach for stabilizing these systems. This delay type is observed in applications such as active suspension systems in vehicles and Direct Current (DC) motor systems [14-17].

Systems with input delays can be controlled using one of two methods: the memory-based (dynamic) approach or the memoryless (static) approach. Static methods are simpler in structure but tend to underperform, whereas dynamic methods, despite their complexity, are more effective at compensating for long delays [18, 19]. In previous papers, numerous techniques have been applied for driving DC motors. In [20], a comprehensive review about the Alternating Current (AC) to DC and DC to DC converters for brushed DC Motor Drives was presented. In this study, different controllers, such as classical Proportional-Integral-Derivative (PID) controllers and intelligent controllers on DC Motor, were investigated. In [21], a robust control approach utilizing the concept of differential flatness was proposed for the bidirectional trajectory tracking of a "full-bridge Buck inverter–DC motor" system. The study demonstrated the effectiveness of this control technique through experimental implementations and simulations, confirming its robustness even under abrupt electrical variations. In [22], a linear parameter varying (LPV) control approach was developed for managing the energy resources of all-electric boats, which included a solar power plant, fuel cell, and battery package. The study designed a robust controller that optimally utilized these energy resources by deriving an LPV representation through sector-nonlinearity techniques. In [23], a unity magnitude shaper command input was proposed for

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damped systems, encompassing both integer and fractional orders. The study demonstrated the analytic design of the unity magnitude shaper and validated its effectiveness through experimental results on a DC motor bench. In [24], the tuning of digital PID controllers for a Controller area network (CAN)-based DC motor was investigated to address the challenges posed by stochastic delays. The study transformed the PID tuning problem into a static-output-feedback controller design for time-delayed systems, utilizing particle swarm optimization (PSO) and linear-quadratic-regulator (LQR) methods. The experimental results validated the effectiveness of the proposed tuning strategy, demonstrating improved performance in managing CAN-induced delays. In this research, time delays were treated as stochastic; nevertheless, these delays did not function as input delays. A recent study [25] proposed an adaptive tracking control strategy for nonlinear systems with input delay and unmeasurable states within predefined sets. Utilizing neural networks and reinforcement learning, this method achieves optimal control and ensures the closed-loop system's semi-global uniform ultimate boundedness, showcasing its potential for practical applications.

Previous research has demonstrated controllers capable of mitigating the effects of delays in the presence of constant disturbances, but these approaches typically relied on trial-and-error numerical tuning without a solid theoretical foundation or Lyapunov stability analysis [16]. The authors of [17] employed the predictor-based H_∞ controller for a first-order DC motor and analyzed its robustness by comparing it with a robust PID controller and a common H_∞ controller, considering time delays of up to 1.1 seconds.

This study introduces a robust memory state feedback controller that guarantees closed-loop stability for a first-order DC motor system under input delays of up to five seconds, utilizing linear matrix inequality (LMI) conditions solved using convex optimization techniques. In addition, an LQR controller has been implemented as a performance evaluation. Compared with a recent work [25], which addressed systems with shorter input delays, the proposed method significantly extends the input delay range while maintaining system stability and robustness. The controller effectively compensates for input delays and attenuates norm-bounded disturbances such as constant, sinusoidal, and stochastic disturbances through a predictive feedback mechanism. Furthermore, compared with the LQR controller, our approach achieves a reduced overshoot, a faster transient response, and superior disturbance attenuation, thereby enhancing the overall system robustness and control precision.

The remainder of this paper is structured as follows: In Section 2, we discuss the theoretical foundations, problem formulation, and derivation of Theorem 1, addressing the challenges associated with time delays and disturbances in DC motor systems. In Section 3, we describe the application of the proposed predictor-based H_∞ controller and introduce the LQR controller as a performance benchmark. This section covers the theoretical framework, numerical simulations demonstrating the effectiveness of the proposed approach under various input delay scenarios and disturbance types, and the implementation of the LQR controller. In Section 4, a comprehensive comparison between the proposed controller and the LQR controller is provided, along with a discussion of the study's limitations and recommendations for future research. Finally, Section 5 concludes the paper by summarizing the key findings on robust DC motor control.

II. PRELIMINARIES AND PROBLEM FORMULATION

Consider a linear system with input delay and norm-bounded disturbance described using

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Bu(t-\tau) + GD(t) \\ &y(t) = Cx(t) \\ &Z(t) = Dx(t) + Eu(t-\tau) \\ &u(t) = \varphi(t),\ t \in [-\tau\ ,\ 0] \end{aligned} \tag{1}$$

Here, $x(t) \in R^n$ is a state vector, $u(t) \in R^m$ is a control input, $y(t) \in R^p$ is a measured output, $Z(t) \in R^d$ is a controlled output, τ is a constant and known delay, $\varphi(t)$ is a continuous initial function on $t \in [-\tau\ .\ 0]$, and $D(t) \in R^q$ is a norm-bounded disturbance. Also, A, B, G, C, D, and E are known real-valued constant matrices with appropriate dimensions [27].

Assumption 1: The exogenous disturbance $D(t)$ is norm-bounded and satisfies the following constraint:

$$\|D(t)\|_{\mathcal{L}_2}^2 = \int_0^\infty \|D(s)\|^2 ds < H \text{ and } H \geq 0 \tag{2}$$

Assumption 2: All system states are accessible or can be measured.

This method aims to minimize the influence of disturbances on system performance by reducing their norm, which is achieved by minimizing the H_∞ norm of the norm-bounded disturbance $D(t)$ relative to the controlled output $Z(t)$. In addition, the adverse effects of input delay are addressed using a predictive vector. To manage the input delay, a prediction vector is computed and utilized to design the control input.

Lemma 1 (Schur complement [26]): Consider the given constant matrices Ω_1, Ω_2, and Ω_2, where $\Omega_1 = \Omega_1^T$, and $\Omega_2 > 0$. Then, the following relation always holds:

$$\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0 \leftrightarrow \begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0 \tag{3}$$

Lemma 2: The time-delay prediction vector for the input of system (1) is obtained from the following equation:

$$\bar{P}(t) = x(t+\tau) = e^{A\tau}x(t) + \int_{t-\tau}^{t} e^{A(t-s)}\,[Bu(s) + GD(s+\tau)]ds \tag{4}$$

Lemma 3: In the absence of norm-bounded disturbances, the prediction vector for system (1) is derived from the following equation:

$$P(t) = x(t+\tau) - \int_{t-\tau}^{t} e^{A(t-s)}\,[GD(s+\tau)]ds = e^{A\tau}x(t) + \int_{t-\tau}^{t} e^{A(t-s)}\,[Bu(s)]ds \tag{5}$$

Subsequently, the robust H_∞ control problem for a closed-loop system is analyzed by leveraging a quadratic Lyapunov function. To provide the necessary conditions for the existence of a delay-dependent memory state-feedback H_∞ controller, the following theorem is presented.

Assuming an initial function, the predictor vector can be determined for subsequent times using a recursive formula derived from Lemmas 2 and 3:

$$\bar{P}(t) = P(t) + e_p(t) \tag{6}$$

where

$$e_p(t) = \int_{t-\tau}^{t} e^{A(t-s)} [GD(s+\tau)]ds \tag{7}$$

Therefore, the predictive-based controller is determined as follows:

$$u(t) = KP(t) = K\bar{P}(t) - Ke_p(t) \tag{8}$$

According to Lemma 2, the delayed input of the system is as follows:

$$u(t-\tau) = Kx(t) - Ke_p(t-\tau) \tag{9}$$

By substituting the delayed input (9) into the system (1), the following results have been obtained:

$$\dot{x}(t) = (A+BK)x(t) + G_1\bar{D}(t) \tag{10}$$

$$Z(t) = (D+EK)x(t) + \bar{E}\bar{D}(t) \tag{11}$$

here,

$$G_1 = [G \;\; B], \qquad \bar{D} = [D^T \quad \widetilde{D}^T]^T$$
$$\widetilde{D}(t) = -Ke_p(t-\tau), \qquad \bar{E} = [0 \quad E] \tag{12}$$

Equation (8) indicates that the prediction vector has been added to the state vector, and its dynamics must be considered. Therefore, the difference equation for the prediction vector can be determined as follows:

$$\dot{P}(t) = e^{A\tau}\dot{x}(t) + Bu(t) - e^{A\tau}Bu(t-\tau) + \int_{t-\tau}^{t} A\, e^{A\tau} [Bu(s)\,]ds \tag{13}$$

By substituting equations (1), (5), and (8) into equation (13), the prediction vector is obtained from the following relation:

$$\dot{P}(t) = (A+BK)P(t) + e^{A\tau}GD(t) = (A+BK)P(t) + G_2\bar{D}(t) \tag{14}$$

where

$$G_2 = M[G \quad 0], \qquad M = e^{A\tau}.$$
$$\bar{D}(t) = [D^T(t) \quad \widetilde{D}^T(t)]^T$$
$$\widetilde{D}(t) = -Ke_p(t-\tau)$$

Definition 1. Given a positive constant γ , the time delay system (1) is said to be robustly asymptotically stable with disturbance damping γ, if the system is asymptotically stable for $\bar{D}(\mathrm{t}) = 0$ and, under zero initial conditions, the following H_∞ criterion is satisfied for all non-zero $\bar{D}(\mathrm{t})$ with bounded norm:

$$\|Z(t)\|_{\mathcal{L}_2} < \gamma\|\bar{D}(t)\|_{\mathcal{L}_2} \tag{15}$$

Theorem 1: A linear system with input delay and bounded-norm disturbance (1) is stable with a predictor-based controller in the presence of external disturbance if, for $D(t) \in \boldsymbol{\mathcal{L}}_2[0 \;\; \infty)$, the criterion $\|Z(t)\|_{\mathcal{L}_2}^2 < \gamma^2\|\bar{D}(t)\|_{\mathcal{L}_2}$ satisfies. Additionally, there exists a positive definite symmetric matrix $X > o$ and an appropriately dimensioned matrix Y, along with constant values γ. λ. L_R, and L_S such that the following linear matrix inequalities are satisfied:

$$\begin{bmatrix} \psi_{11} & 0 & G & B & XD^T + Y^TE^T \\ 0 & \psi_{22} & \lambda HG & 0 & 0 \\ G^T & \lambda G^TH^T & -\gamma^2 I & 0 & 0 \\ B^T & 0 & 0 & -\gamma^2 I & E^T \\ DX+EY & 0 & 0 & E & -I \end{bmatrix} < 0 \tag{16}$$

$$\begin{bmatrix} L_R I & Y^T \\ Y & I \end{bmatrix} > 0 \tag{17}$$

$$\begin{bmatrix} L_S I & I \\ I & X \end{bmatrix} > 0 \tag{18}$$

where, $\Psi_{11} = XA^T + Y^TB^T + AX + BY$ and $\Psi_{22} = \lambda(XA^T + Y^TB^T + AX + BY)$

The suitable gain for the predictor-based controller, as given in (8), can be determined from the following equation:

$$K = YX^{-1} \tag{19}$$

Proof. To analyze and prove Theorem 1, a quadratic Lyapunov function has been employed, as defined by the following equation:

$$V(t) \triangleq V_1(x(t)) + V_2(P(t)) = x^T(t)Q_1x(t\,) + P^T(t)Q_2P(t) \tag{20}$$

where, Q_1 and Q_2 are positive definite matrices.

By leveraging Lemmas (1) to (3) along with the quadratic Lyapunov function in Equation (20), the proof of Theorem 1 is derived. Notably, further details regarding this method can be found in [27].

III . IMPLEMENTION, SIIMULATION, AND VALIDATION FOR DC MOTOR CONTROL

A.Utilization of DC Motors

In this subsection, a robust controller is implemented on a first-order DC motor model. The simplified transfer function of a first-order DC motor with input delay is represented by the following equation:

$$T(s) = \frac{k}{1+sT} e^{-\tau s} \tag{21}$$

Here, the transfer function's output is the angular velocity (ω) and the input is the applied voltage u. In Equation (21), $T(s)$ denotes the DC motor's transfer function, where k represents the steady-state gain, T is the time constant, and τ is the input delay. The transfer function in Equation (36) provides a classical simplified model of small DC motors, where the inductance is typically neglected. The term $e^{-\tau s}$ accounts for the presence of input delay.
Therefore, the state-space model of this system, incorporating the norm-bounded disturbance (d), can be represented as follows [16]:

$$\dot{\omega} = a\omega + bu(t-\tau) + d \tag{22}$$

where $a = -\frac{1}{T}$, and $b = \frac{K}{T}$. The parameter values for $k, T,\ \omega_{ref}$ are given as 177.75, 1.14 s, and 150 rad/s, respectively.

For the simulation procedure, the problem is initially transformed from tracking to regulation by applying certain variable changes, after which an H_∞ robust controller is designed. The following variable transformations are introduced for system (37):

$$\begin{cases} x = \omega - \omega_{ref} \rightarrow \dot{x} = \dot{\omega} \\ v = u - u_{ref} \end{cases} \tag{23}$$

Thus, by substituting Equation (38) into Equation (37), the following equations are obtained:

$$\dot{x} = a(x + \omega_{ref}) + b(v(t-\tau) + u_{ref}) + d \tag{24}$$

$$\dot{x} = ax + bv(t-\tau) + a\omega_{ref} + bu_{ref} + d \tag{25}$$

Given the relation $u_{ref} = -\frac{a}{b}\omega_{ref}$, the equations can be rewritten as follows:

$$\begin{cases} \dot{x} = ax + bv(t-\tau) + d \\ Z = Cx + Ev(t-\tau) \end{cases} \tag{26}$$

Using Theorem 1, the design of an H_∞ memory-state feedback controller is expressed as $u(t) = \mathrm{K}P(t)$, where $P(t)$ is the prediction vector for the system, and K is the state feedback gain matrix. The matrix K must be designed to ensure that the closed-loop system (without disturbance) remains asymptotically stable. Furthermore, the closed-loop stability under a disturbance is guaranteed by setting a positive constant γ and assuming zero initial conditions, such that the H_∞ criterion, represented by Equation (15), holds for all nonzero norm-bounded disturbances. Therefore, the feedback gain matrix K is determined by solving the linear matrix inequalities (LMIs) in Theorem 1, and the prediction vector is given by the following equation:

$$P(t) = e^{A\tau}x(t) + \int_{t-\tau}^{t} e^{A(t-s)}\,[Bu(s)]ds \tag{27}$$

Consequently, the controller design for system (22) is carried out as follows:

$$v = KP = Ke^{A\tau}x(t) + K\int_{t-\tau}^{t} e^{A(t-s)}\,[Bv(s)]ds \tag{28}$$

By substituting $v = u - u_{ref}$ into Equation (28), the final form of the controller is given as follows:

$$u = u_{ref} + Ke^{A\tau}(\omega - \omega_{ref}) + K\int_{t-\tau}^{t} e^{A(t-s)}\left[(Bu(s) - u_{ref}))\right]ds \tag{29}$$

In the following subsection, an LQR controller is employed to facilitate a comparative analysis between the proposed method and its performance in a first-order DC motor model with delay. Subsequently, a detailed comparison of the two controllers is presented.

B. LQR Controller

The Linear Quadratic Regulator (LQR) is a fundamental method in control theory used to design controllers for dynamic systems. This method aims to determine the optimal control inputs that minimize a cost function, typically a quadratic function representing a trade-off between the system's state deviations and control effort. The LQR approach involves solving a Riccati differential equation to derive the optimal feedback gains. These gains, when applied, ensure that the system remains stable and performs optimally in the presence of disturbances and uncertainties [28].
An LQR controller is designed as following:

$$u_{LQR}(t) = K_{LQR}(x(t) - \omega_{ref}) + u_{ref} \tag{30}$$

In equation (30), K_{LQR} is the LQR gain, which is calculated by solving the Riccati equation for the given system dynamics and weighting matrices. The resulting gain is then used to compute the optimal control input in the simulation. By leveraging MATLAB's lqr function, we efficiently computed the optimal LQR gain for our DC motor system, ensuring high-performance control.

In comparing the behavior of the proposed Controller with the LQR Controller, several significant differences in performance have been observed.

C. Simulation and Comparative Analysis of the Proposed Controller and the LQR Controller

In this subsection, we examine the results of applying a robust state feedback controller, based on predictive strategies and LQR controller, to a first-order DC motor. To assess the controller's performance under time delays exceeding one second, we compare norm-bounded disturbances in three forms: time-invariant, sinusoidal, and stochastic.

First, the feedback gain K is determined through linear matrix inequalities (LMIs) as derived in Theorem 1, using the YALMIP toolbox within MATLAB. Subsequently, simulations in MATLAB are conducted with the obtained gain K to evaluate the controller's effectiveness in compensating for input delay and attenuating disturbances.

The parameters for determining the gain K and the simulators are considered as follows [16]

$$\gamma = 0.3, \lambda = 0.01, L_R = 10^3, L_s = 10^3, c = 0.7, dt = 0.01 \quad (31)$$

In previous research, no specific technique was employed for determining the gain K; rather, a trial-and-error approach was often used. Additionally, prior studies considered time delays up to one second. However, in this paper, the optimal gain K is obtained using the LMI method and convex optimization for input delays up to five seconds, thereby reducing conservatism by employing a delay-dependent controller.

IV. TIME-INVARIANT NORM-BOUNDED DISTURBANCE

In this section, a time-invariant norm-bounded disturbance with $d = 24\frac{rad}{s}$ is applied during the interval from the tenth to the thirtieth second, with input delays 1.5 to 5 seconds. The feedback gain obtained using the LMI method is $K = -2.4128$ for an input delay of 1.5 seconds and $K = -2.4127$ for the other input delay values. Fig. s 1 to 5 illustrate the effect of input delay variations including 1.5, 2, 3, 4, and 5 seconds, respectively, on the system.

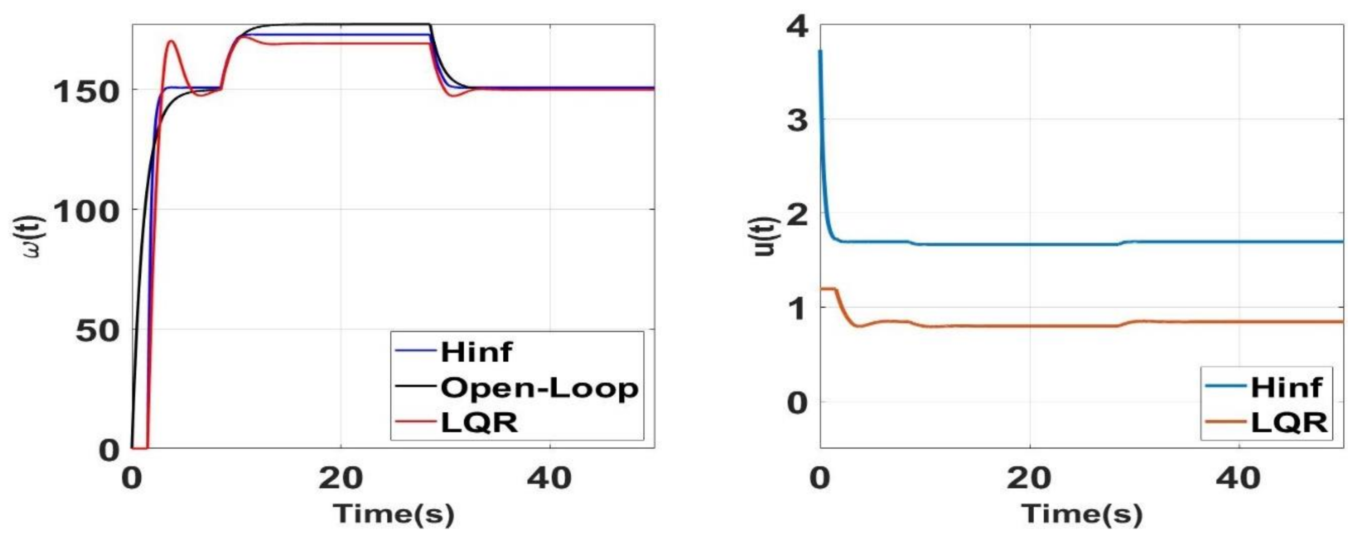

Fig. 1: Angular velocity and control input for an input delay of 1.5 seconds under a constant norm-bounded disturbance using the proposed controller and the LQR controller

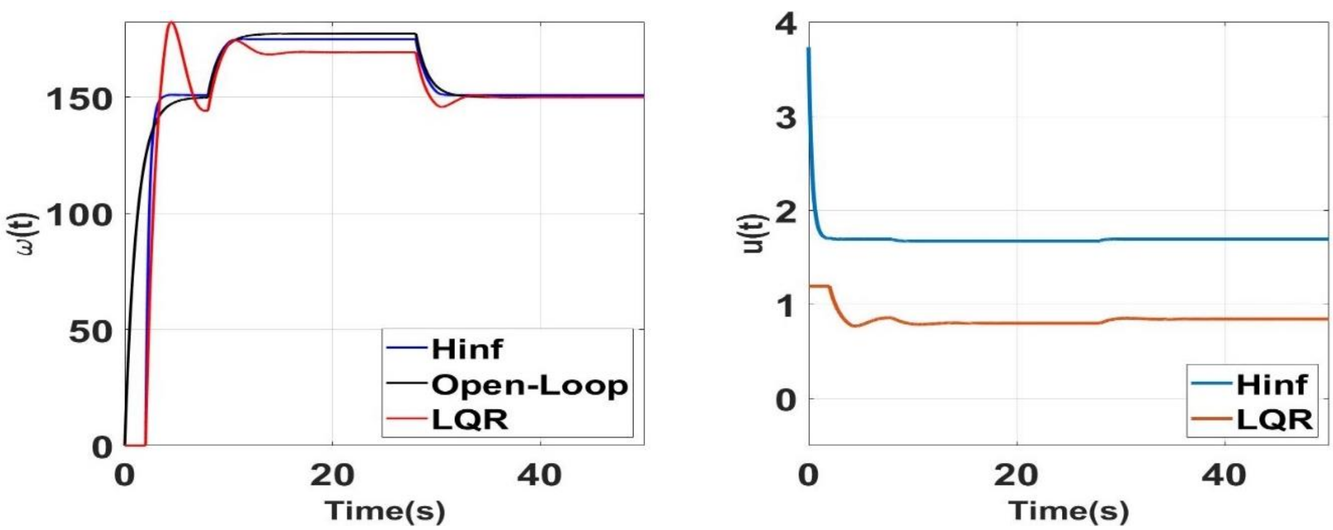

Fig. 2: Angular velocity and control input for an input delay of 2 seconds under a constant norm-bounded disturbance using the proposed controller and the LQR controller

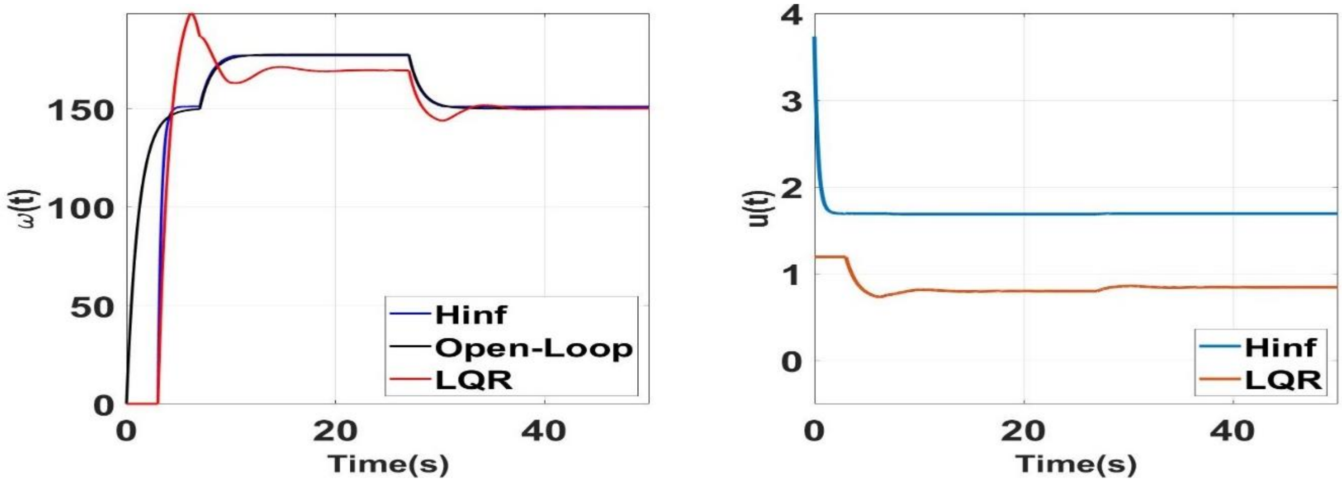

Fig. 3: Angular velocity and control input for an input delay of 3 seconds under a constant norm-bounded disturbance using the proposed controller and the LQR controller

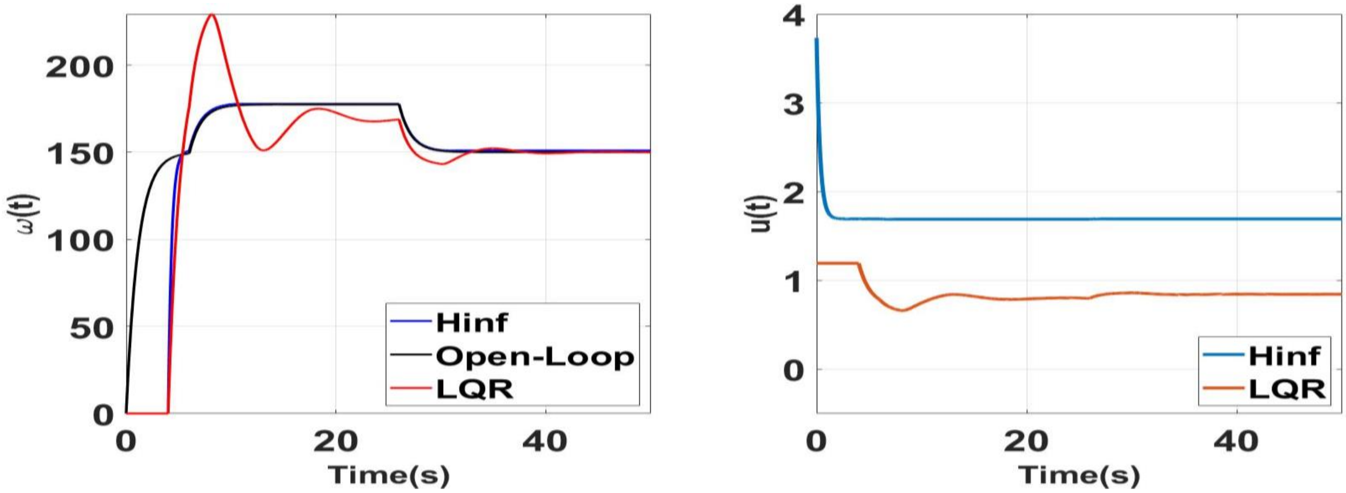

Fig. 4: Angular velocity and control input for an input delay of 4 seconds under a constant norm-bounded disturbance using the proposed controller and the LQR controller

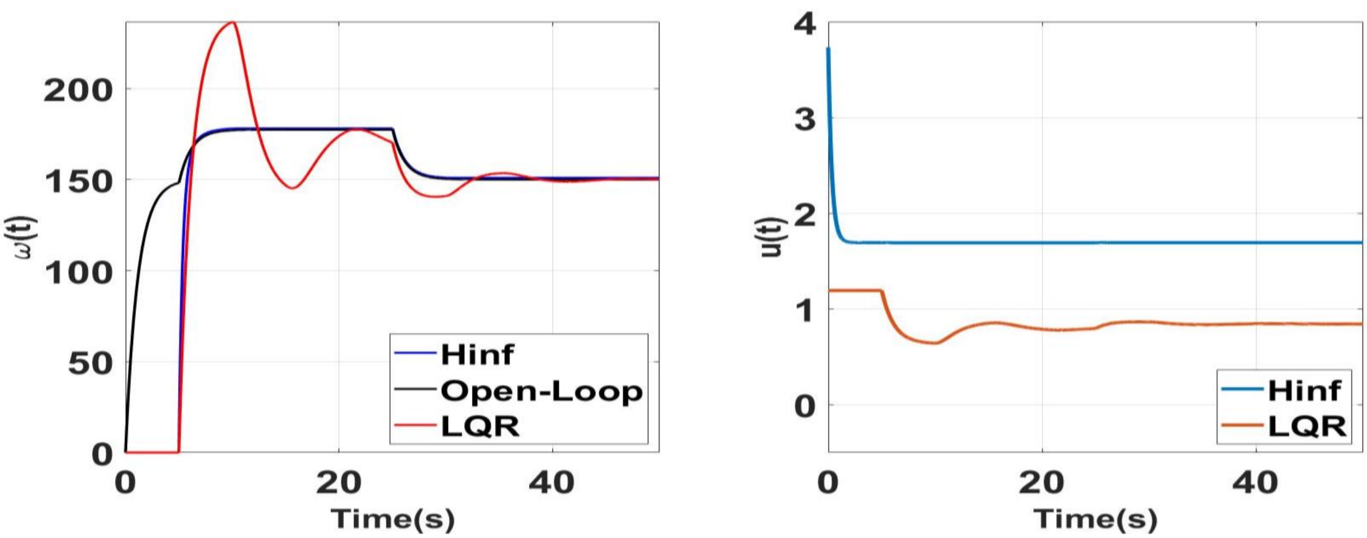

Fig. 5: Angular velocity and control input for an input delay of 5 seconds under a constant norm-bounded disturbance using the proposed controller and the LQR controller

The stability of the closed-loop system is maintained for input delays ranging from 1.5 to 5 seconds. Furthermore, the controller effectively attenuates the norm-bounded disturbance within this range. As the delay increases, the level of disturbance attenuation gradually decreases, and at a delay of $\tau = 5$ seconds, the disturbance attenuation is completely lost. Therefore, it can be noted that the performance and speed of the system in closed-loop mode with the robust controller are superior to those of the open-loop system, even in the presence of delay.

Sinusoidal Norm-Bounded Disturbance:

In this section, the system is subjected to a norm-bounded sinusoidal disturbance of the form $D(t)=5+10\sin(0.3t)$. In this case, the stability of the closed-loop system is guaranteed in the presence of input delays ranging from 1.5 to 5 seconds, and it effectively achieves disturbance attenuation. Fig. s 6 through 10 demonstrate the impact of varying input delays of 1.5, 2, 3, 4, and 5 seconds on the system in the presence of a sinusoidal norm-bounded disturbance.

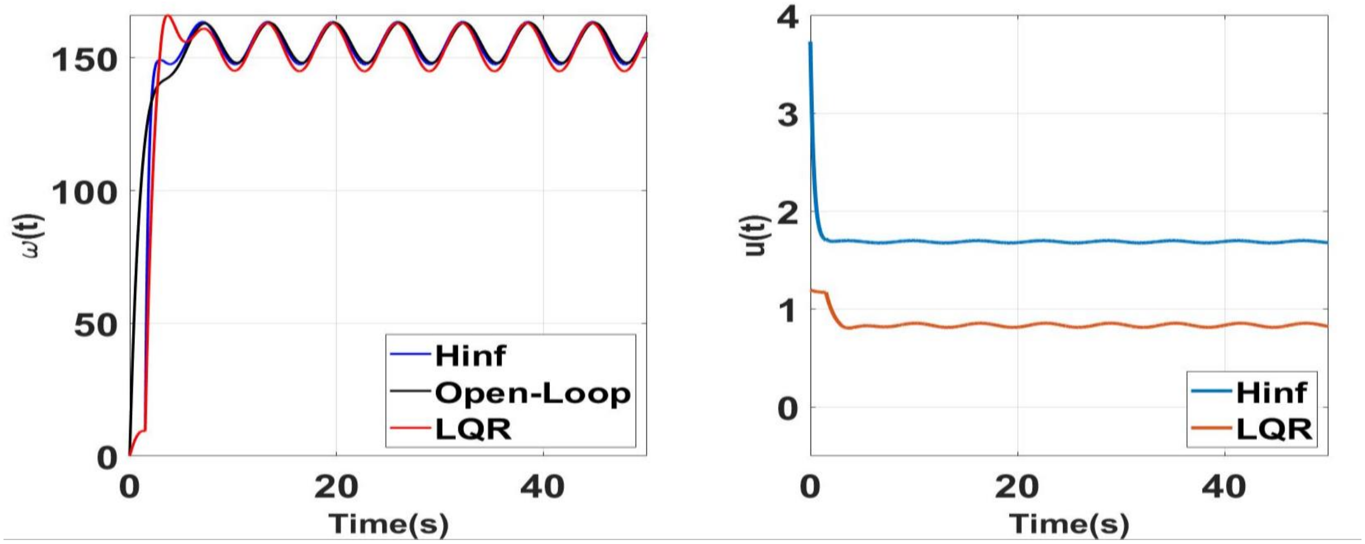

Fig. 6: Angular velocity and control input for an input delay of 1.5 seconds under a sinusoidal norm-bounded disturbance using the proposed controller and the LQR controller

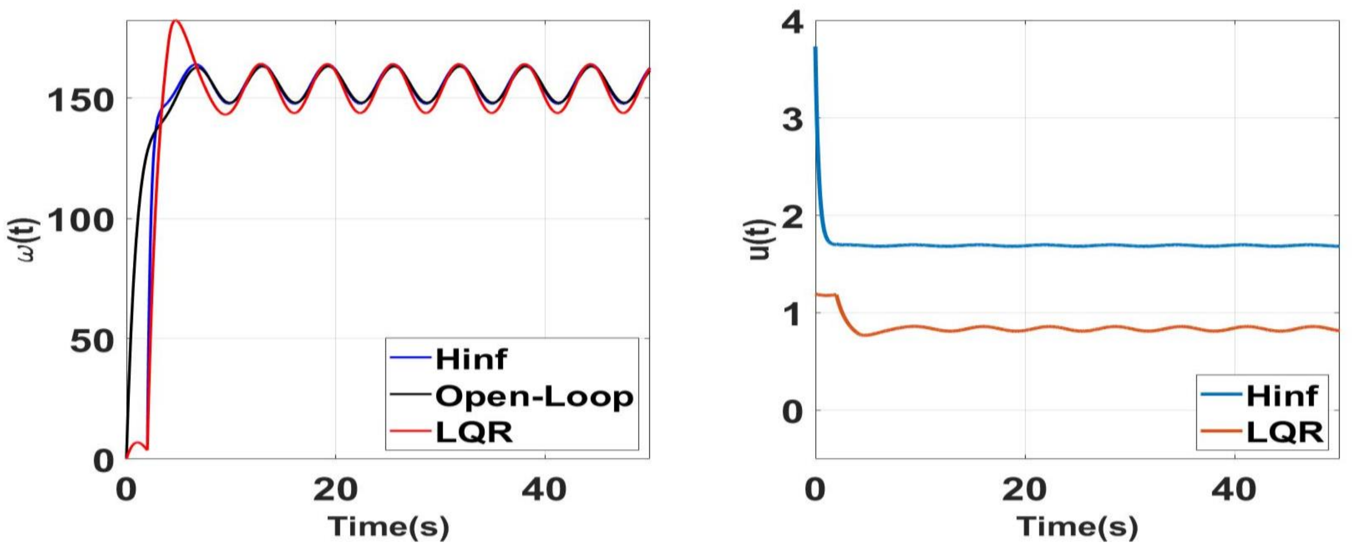

Fig. 7: Angular velocity and control input for an input delay of 2 seconds under a sinusoidal norm-bounded disturbance using the proposed controller and the LQR controller

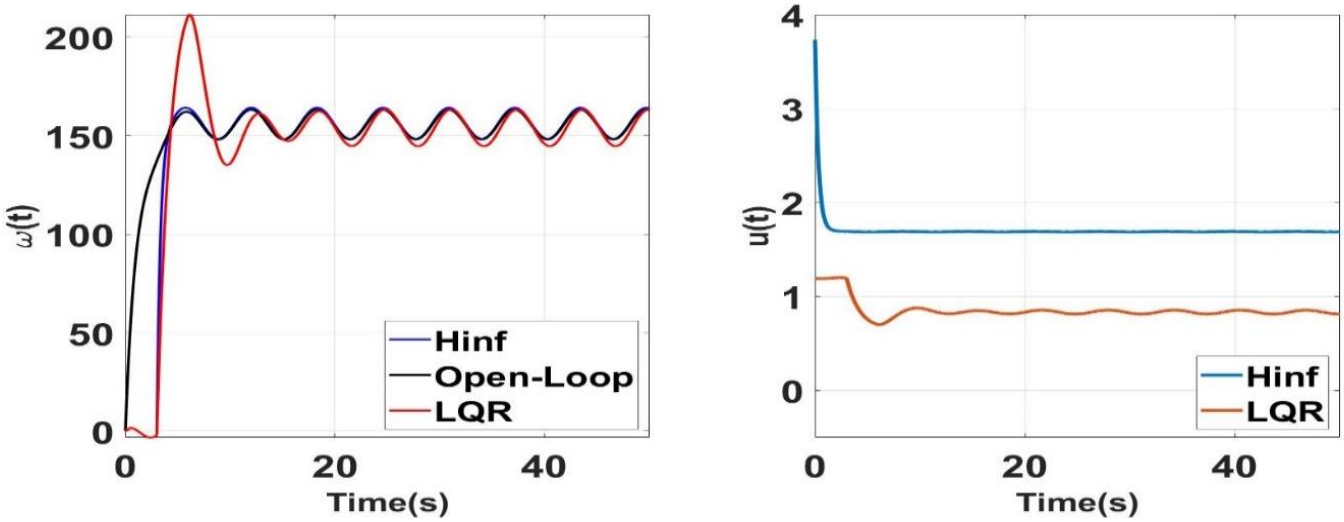

Fig. 8: Angular velocity and control input for an input delay of 3 seconds under a sinusoidal norm-bounded disturbance using the proposed controller and the LQR controller

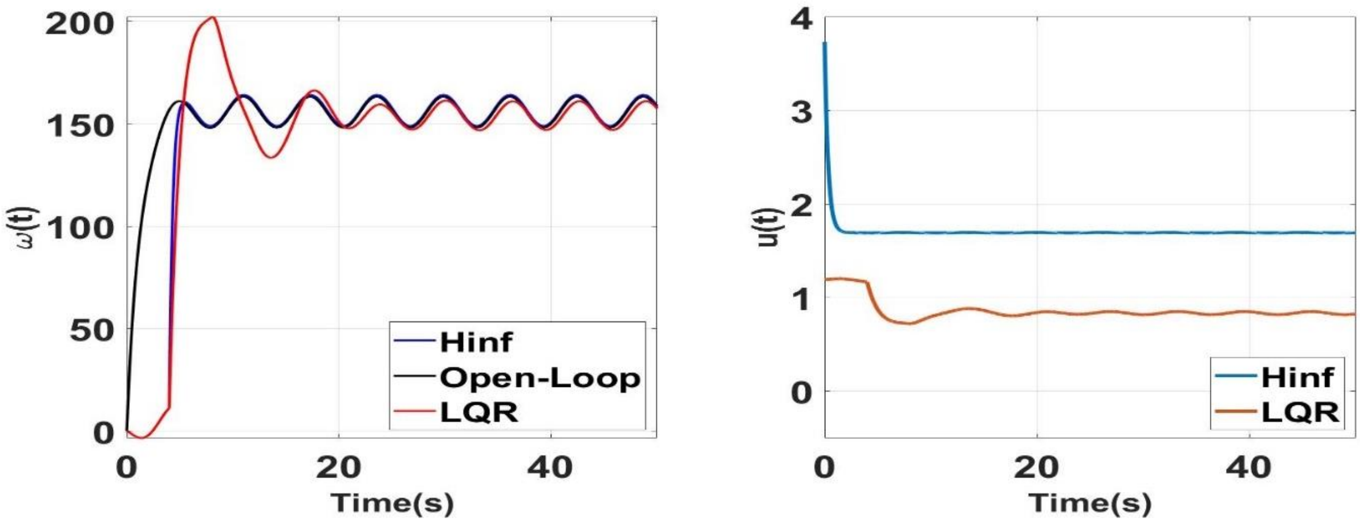

Fig. 9: Angular velocity and control input for an input delay of 4 seconds under a sinusoidal norm-bounded disturbance using the proposed controller and the LQR controller

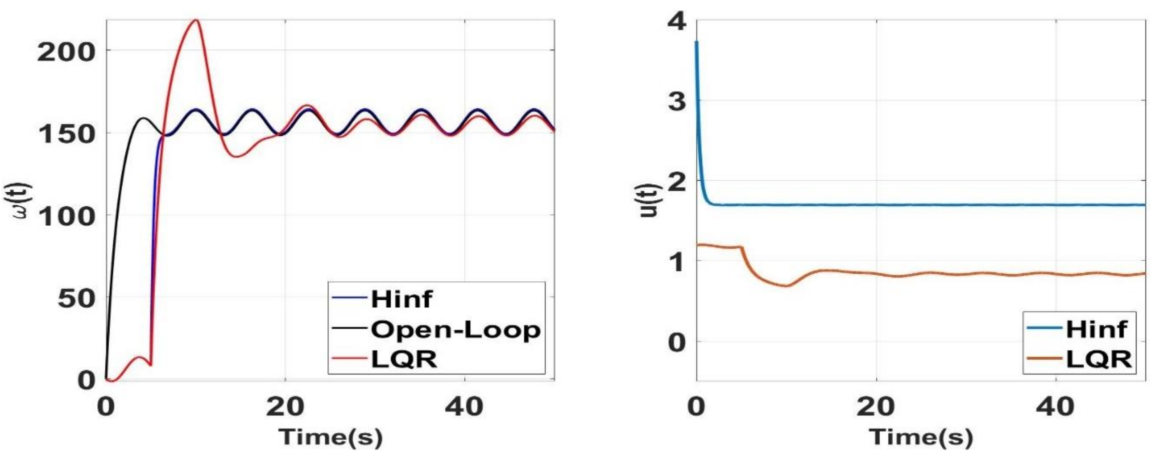

Fig. 10: Angular velocity and control input for an input delay of 5 seconds under a sinusoidal norm-bounded disturbance using the proposed controller and the LQR controller

V. STOCHASTIC NORM-BOUNDED DISTURBANCE

This section examines the performance of robust controllers when the system is subjected to stochastic norm-bounded disturbances. The stochastic disturbance utilized in this analysis is depicted in Fig. 11.

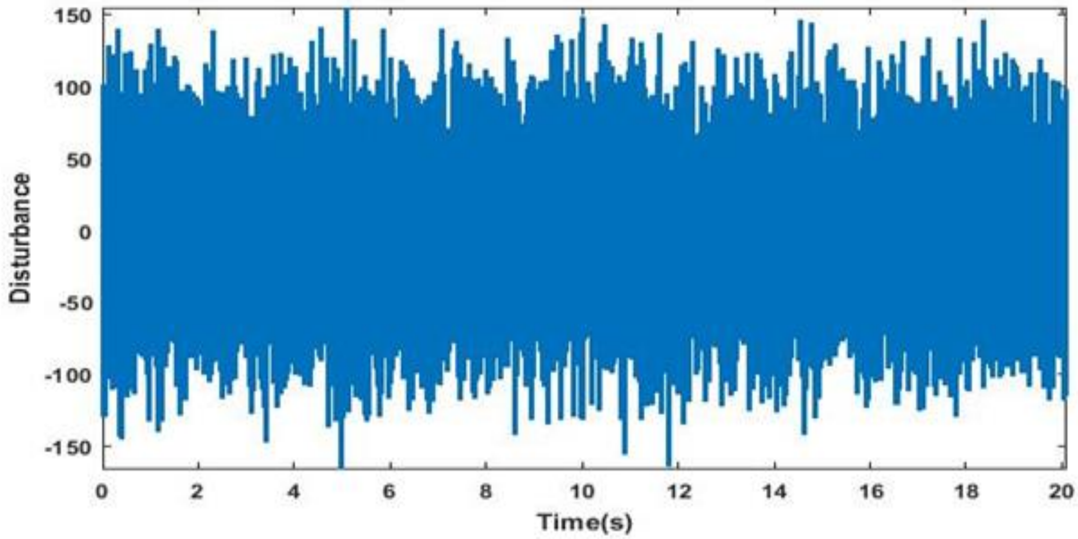

Fig. 11: Stochastic norm-bounded Disturbance

In this scenario, the stability of the closed-loop system is assured despite input delays between 1.5 and 5 seconds, and it successfully mitigates disturbances. Fig. s 12 to 16 illustrate how different input delays of 1.5, 2, 3, 4, and 5 seconds affect the system when subjected to a stochastic norm-bounded disturbance.

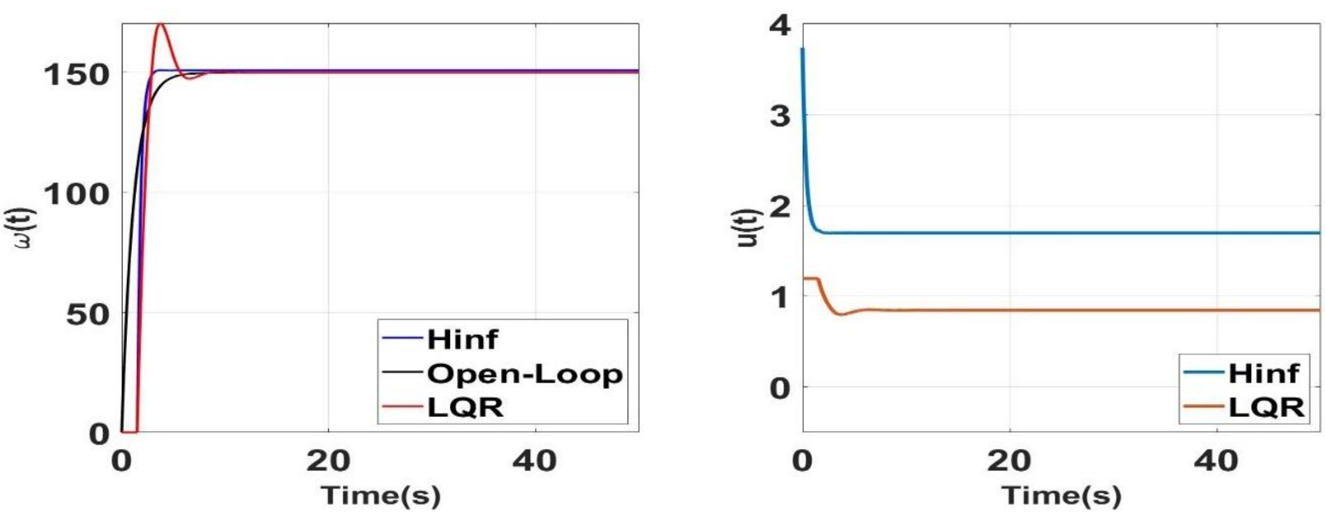

Fig. 12: Angular velocity and control input for an input delay of 1.5 seconds under a stochastic norm-bounded disturbance using the proposed controller and the LQR controller

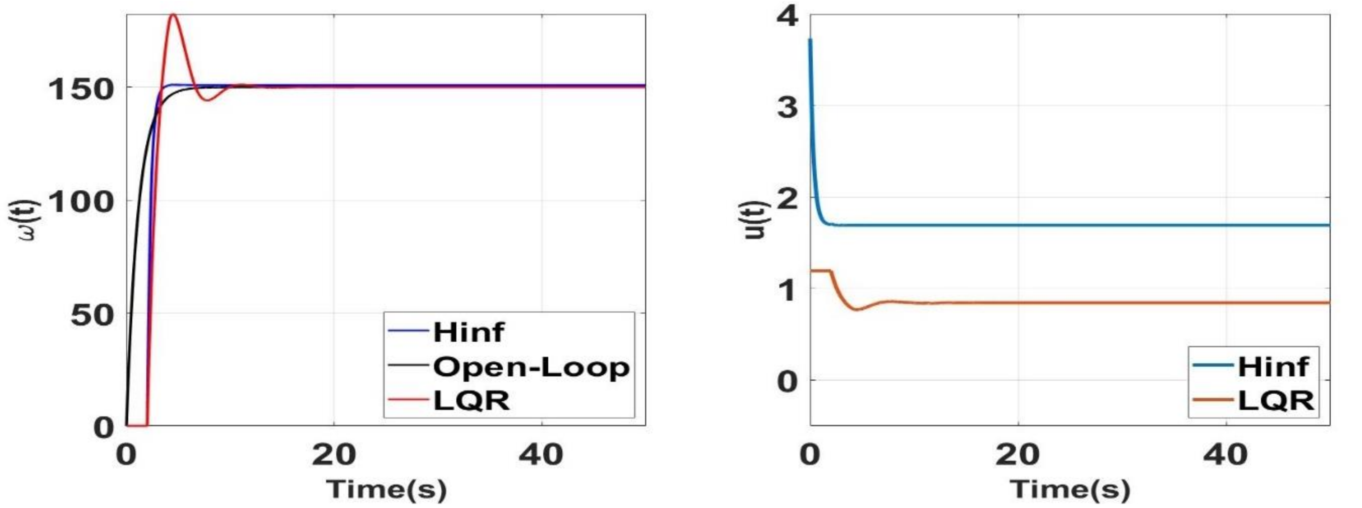

Fig. 13: Angular velocity and control input for an input delay of 2 seconds under a stochastic norm-bounded disturbance using the proposed controller and the LQR controller

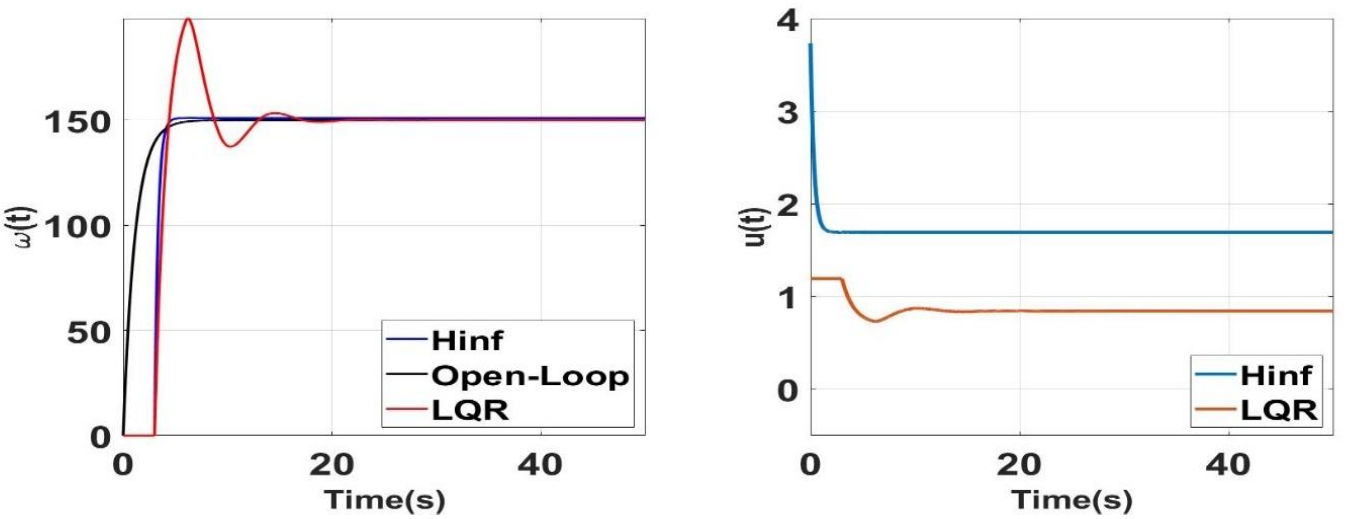

Fig. 14: Angular velocity and control input for an input delay of 3 seconds under a stochastic norm-bounded disturbance using the proposed controller and the LQR controller

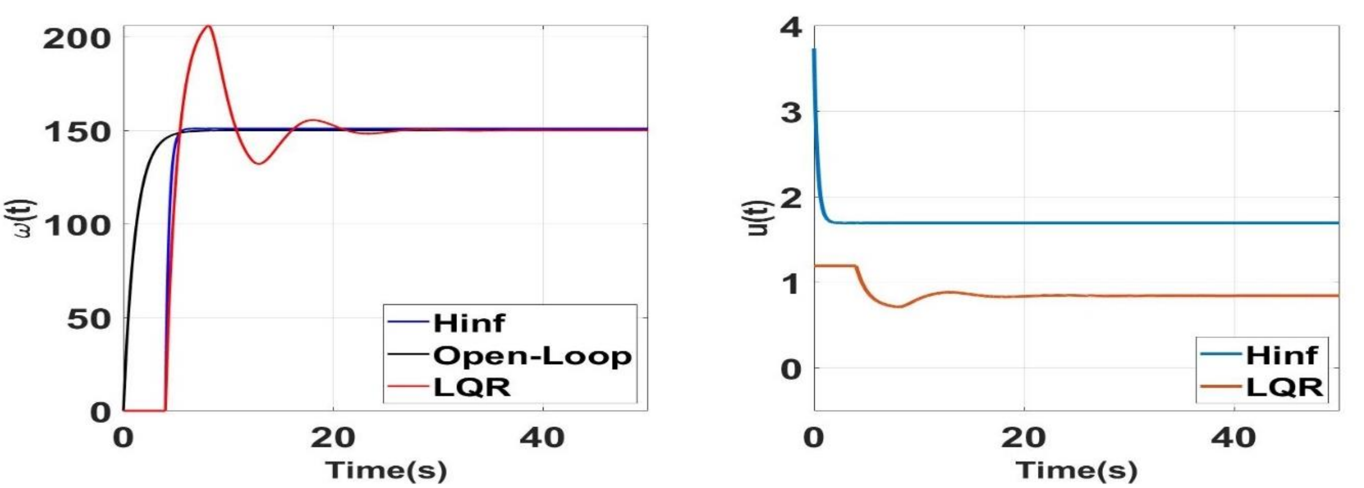

Fig. 15: Angular velocity and control input for an input delay of 4 seconds under a stochastic norm-bounded disturbance using the proposed controller and the LQR controller

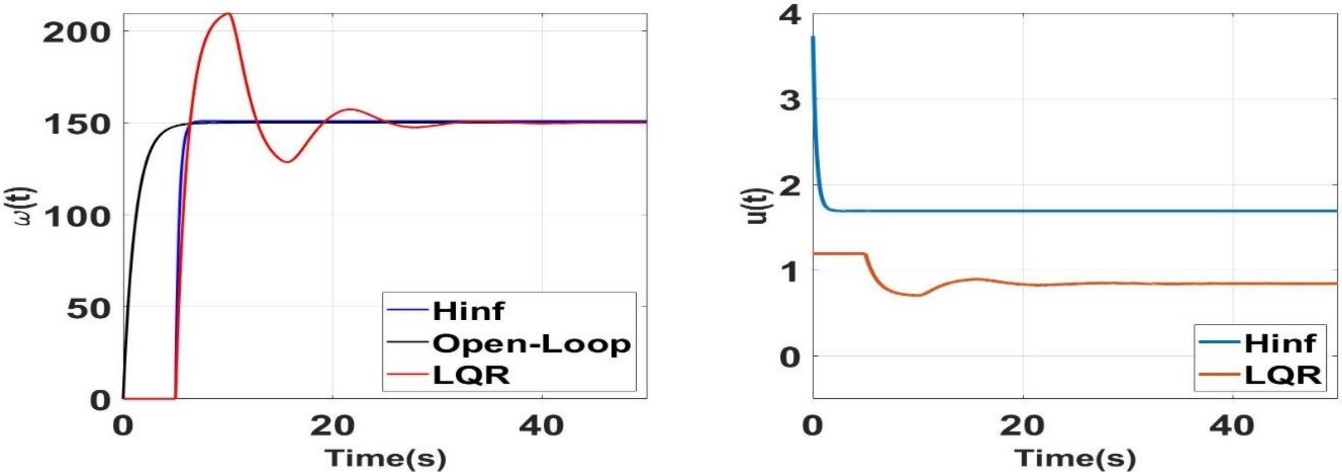

Fig. 16: Angular velocity and control input for an input delay of 5 seconds under a stochastic norm-bounded disturbance using the proposed controller and the LQR controller

Fig. s 12 to 16 demonstrate that the proposed controller delivered satisfactory outcomes, effectively eliminating the norm-bounded disturbance and mitigating the impact of time delays with high precision.

VI . DISCUSSION AND LIMITATIONS OF PROPOSED METHOD

A. Comparison between the Proposed Controller and the LQR Controller

As depicted in the Fig. s of Subsection 3.3, the angular velocity and control input in the delayed system exhibit superior performance under the proposed controller compared to the LQR controller across three disturbance types: constant, sinusoidal, and random. To quantitatively assess these controllers, Tables I through III present a comparative analysis based on settling time and overshoot metrics for constant, sinusoidal, and stochastic disturbances, respectively.

TABLE I

Comparison of the Performance of the Robust and LQR Controllers Under a Constant Norm-Bounded Disturbance

Controller	**Criteria** Settling Time (seconds)	Overshoot (%)
Robust Controller	10.73	18.70
LQR Controller	11.09	57.75

TABLE II

Comparison of the Performance of the Robust and LQR Controllers Under a Sinusoidal Norm-Bounded Disturbance

Controller	**Criteria** Settling Time (seconds)	Overshoot (%)
Robust Controller	11.21	9.39
LQR Controller	11.36	45.75

TABLE III

Comparison of the Performance of the Robust and LQR Controllers Under a Stochastic Norm-Bounded Disturbance

Controller	**Criteria** Settling Time (seconds)	Overshoot (%)
Robust Controller	11.19	0.65
LQR Controller	11.35	39.73

Based on the results presented in the Fig. s of Subsection 3.3 and the data summarized in Tables I through III, a comparison between the robust controller and the LQR controller reveals several significant performance differences. These differences will be discussed in detail below:

A. **Performance under All Conditions:** The predictive-based H_{∞} Controller consistently outperforms the LQR Controller across various conditions, such as various time delays and various disturbances. One of the critical performance metrics, the settling time, is smaller for the predictive-based H_{∞} Controller. This indicates that the system reaches its desired steady state more quickly with less oscillation or deviation. Additionally, the overshoot, which is the extent to which the system exceeds its target value before settling, is also considerably smaller for the predictive-based H_{∞} Controller.

- **Response to Reference Speed:** The predictive-based H_{∞} Controller exhibits the capability to reach the reference speed in a significantly shorter time compared to the LQR Controller. This rapid convergence to the desired speed ensures that the system operates efficiently and meets the performance expectations promptly.
- **Time-Delay Robustness:** One of the standouts features of the predictive-based H_{∞} Controller is its robustness in the presence of time delays. When subjected to time delays, this controller can effectively reject nearly all the adverse effects, maintaining system stability. Even when

confronted with substantial time delays, such as 4 or 5 seconds, the predictive-based H_∞ Controller demonstrates significant resilience and stability. This robustness ensures that the system continues to perform optimally without being significantly affected by delays.

- **Smoothness of Control Input:** The control input provided by the predictive-based H_∞ Controller is markedly smoother than that of the LQR Controller. This smooth control input translates to improving the overall lifespan and reliability of the system. The reduced fluctuations and smoother transitions also contribute to enhanced performance and efficiency.

In summary, the predictive-based H_∞ Controller offers superior performance over the LQR Controller in various aspects, including faster settling times, reduced overshoot, robustness against time delays, and smoother control inputs. These attributes make the predictive-based H_∞ Controller a more effective and reliable choice for controlling DC motor.

B, Impact of Gamma Variations on Robust Controller Performance

In this subsection, the performance of the robust controller under varying gamma values is evaluated. Table IV presents the settling time and memory state feedback gain (K) corresponding to different gamma values. As observed, among the gamma values of 0.3, 1, and 2, the case of $\gamma = 0.3$ exhibits the best performance, with the shortest settling time and the lowest control effort.

TABLE IV
Evaluation of the Robust Controller's Performance Under Gamma Variations

Robust Controller	**Gamma Variations** γ=0.3	γ=1	γ=2
Settling Time (second)	11.19	11.64	11.68
K	-2.41	-1.52	-1.46

C. Limitation of the Proposed Method

In practical applications, not all state variables of a system can be directly measured due to inherent sensor limitations and increased cost. Although the proposed robust predictor-based controller was originally developed under the assumption of full state availability, real-world implementations require the integration of state estimation techniques to overcome these constraints. Moreover, it should be noted that the delay considered in this study is assumed to be constant; however, future research may extend the analysis to accommodate time-varying delays, thereby enabling a comprehensive evaluation of closed-loop stability under more realistic operating conditions. This extended investigation could facilitate the development of integrated observer-controller designs that effectively address both state estimation challenges and dynamic delay variations, ultimately enhancing the overall performance and robustness of the control system. The limitations and challenges of this research, along with potential directions for further advancement and suggestions for future studies, are discussed in the following:

- **Observer-Based Techniques:** Observer-based techniques, such as employing a Luenberger observer or a Kalman filter ([29] and [30]), can be effectively used to estimate unmeasured states from available output measurements. However, incorporating these observers into the robust predictor-based control framework in the time delay system presents challenges due to the interdependence between observer and controller dynamics. Precise tuning of the observer gain is essential to achieve rapid and accurate convergence of the estimation error, as any mismatch between the observer model and the actual system dynamics can significantly degrade performance. Future work could focus on developing integrated observer-controller designs that explicitly address these complexities.
- **Time-Varying Delays:** In this study, the input delay is assumed to be constant. Extending the analysis to accommodate time-varying delays would allow for a more comprehensive evaluation of closed-loop stability under dynamic conditions. Given the inherent complexity of incorporating time-varying delays, dedicated investigation is warranted. Future research should explore delay-dependent robust observer designs, adaptive filtering techniques, or artificial intelligence-based estimation methods to enhance control performance in scenarios where delays vary with time.

In the present study, the robust controller is designed using LMIs solved via MATLAB's YALMIP toolbox, demonstrating computational feasibility for the first-order DC motor system. However, for higher-order or more complex systems, the computational burden may increase significantly, potentially necessitating advanced optimization techniques or parallel computing strategies to achieve real-time performance. Moreover, practical implementations face additional challenges such as sensor noise, and unmodeled dynamics that can affect controller performance. Future research should focus on experimental validation and the development of robust, real-time computational methods to address these issues.

VII. Conclusion

In this paper, we propose a robust memory state feedback controller that is specifically designed to address input delays. By employing linear matrix inequality (LMI) conditions, we ensure the closed-loop stability of a first-order DC motor system. Our research focuses on designing a robust controller for the first-order DC motor while accounting for input delays and norm-bounded disturbances across three scenarios: time-invariant, sinusoidal, and stochastic disturbances. The findings indicate that by implementing a predictive vector, we can effectively eliminate time delays from the input by extending it up to five seconds—five times longer than what previous studies have achieved. Moreover, the application of our robust controller, we successfully dampen the effects of norm-bounded disturbances across all three scenarios.

Furthermore, comparative analysis with an LQR controller reveals that our proposed robust controller consistently achieves lower overshoot, faster transient response, and superior disturbance attenuation, underscoring its enhanced performance over traditional LQR-based strategies.

While our study assumes a constant input delay, in practical applications, delays may vary over time, and not all state variables can be directly measured due to sensor limitations and increased cost. This necessitates the integration of state estimation techniques, such as observer-based methods, into the control framework. Extending the analysis to accommodate time-varying delays and developing integrated observer-controller designs represent promising directions for future research, enabling a comprehensive evaluation of closed-loop stability under realistic operating conditions. Future work will focus on extending the proposed approach to systems with time-varying delays, and exploring integrated observer-controller designs to enhance its applicability in practical scenarios.

REFERENCES

[1] C. Hua, L. Zhang, and X. Guan, Robust Control for Nonlinear Time-Delay Systems, Springer, 2018, https://doi.org/10.1007/978-981-10-5131-9.

[2] H. Chen, X. Long, Y. Tang, and R. Xu, "Passive and H∞ control based on non-fragile observer for a class of uncertain nonlinear systems with input time-delay," Journal of Vibration and Control, Aug. 2023, https://doi.org/10.1177/10775463231193458.

[3] A. K. Ali and M. M. Mahmoud, "Improved Design of Nonlinear Control Systems with Time Delay," International Journal of Robotics and Control Systems, vol. 2, no. 2, pp. 317–331, May 2022, https://doi.org/10.31763/ijrcs.v2i2.631.

[4] L. Pekař, P. Navrátil, and R. Matušů, "Some Recent Results on Direct Delay-Dependent Stability Analysis: Review and Open Problems," in Advances in intelligent systems and computing, 2018, pp. 25–34. https://doi.org/10.1007/978-3-319-91192-2_3.

[5] T.-Y. Cai and C. Hwang, "A Simple Procedure for the Creation of Stability Charts in Delay Parameter Space for a Class of LTI Systems with Two Delays," IEEE Access, vol. 11, pp. 23053–23062, Jan. 2023, https://doi.org/10.1109/access.2023.3248951.

[6] J. Chen, J. H. Park, S. Xu, and B. Zhang, "A survey of inequality techniques for stability analysis of time-delay systems," International Journal of Robust and Nonlinear Control, vol. 32, no. 11, pp. 6412–6440, Apr. 2022, https://doi.org/10.1002/rnc.6151.

[7] X. Zhang, X. Lu, and Z. Liu, "Razumikhin and Krasovskii methods for asymptotic stability of nonlinear delay impulsive systems on time scales," Nonlinear Analysis Hybrid Systems, vol. 32, pp. 1–9, Nov. 2018, https://doi.org/10.1016/j.nahs.2018.10.010.

[8] B. Zhou and A. V. Egorov, "Razumikhin and Krasovskii stability theorems for time-varying time-delay systems," Automatica, vol. 71, pp. 281–291, Jun. 2016, https://doi.org/10.1016/j.automatica.2016.04.048.

[9] Y. Zhang, L. Xie, X. Xie, Z.-Y. Sun, and K. Zhang, "Fuzzy Adaptive Control for Stochastic Nonstrict Feedback Systems with Multiple Time-Delays: A Novel Lyapunov-Krasovskii Method," IEEE Transactions on Fuzzy Systems, vol. 32, no. 6, pp. 3815–3824, Apr. 2024, https://doi.org/10.1109/tfuzz.2024.3384588.

[10] A. Pandey and D. M. Adhyaru, "Stability Analysis of Electromagnetic Levitation System Using Lyapunov-Krasovskii's Method," 2022 International Conference for Advancement in Technology (ICONAT), Jan. 2022, https://doi.org/10.1109/iconat53423.2022.9726080.

[11] E. Moradi, "Finite time stabilization of time-delay nonlinear systems with uncertainty and time-varying delay," Journal of Control, vol. 14, no. 2, pp. 79–87, Jun. 2020, https://doi.org/10.29252/joc.14.2.79.

[12] E. Moradi, M. R. Jahed-Motlagh, and M. B. Yazdi, "Delay-Dependent Finite-Time Stabilization of Uncertain Switched Time-Delay Systems with Norm-Bounded Disturbance," IETE Journal of Research, vol. 64, no. 2, pp. 195–208, Aug. 2017, https://doi.org/10.1080/03772063.2017.1351898.

[13] S. Hao, T. Liu, and B. Zhou, "Predictor-based output feedback control design for sampled systems with input delay subject to disturbance," IET Control Theory and Applications, vol. 11, no. 18, pp. 3329–3340, Sep. 2017, https://doi.org/10.1049/iet-cta.2017.0504.

[14] H. D. Choi, C. K. Ahn, M. T. Lim, and M. K. Song, "Dynamic output-feedback H ∞ control for active half-vehicle suspension systems with time-varying input delay," International Journal of Control Automation and Systems, vol. 14, no. 1, pp. 59–68, Feb. 2016, https://doi.org/10.1007/s12555-015-2005-8.

[15] X. Jin, J. Wang, Z. Yan, L. Xu, G. Yin, and N. Chen, "Robust Vibration Control for Active Suspension System of In-Wheel-Motor-Driven Electric Vehicle Via μ-Synthesis Methodology," Journal of Dynamic Systems Measurement and Control, vol. 144, no. 5, Jan. 2022, https://doi.org/10.1115/1.4053661.

[16] V. L´echapp´e and O. Salas Jes´us de Le´on, "Predictive control of disturbed systems with input delay: experimental validation on a DC motor," IFAC-PapersOnLine, pp. 292–297, 2015, https://doi.org/10.1016/j.ifacol.2015.09.393.

[17] N. Maleki and E. Moradi, "Robust H∞ Control of DC Motor in the Presence of Input Delay and Disturbance by the Predictor-Based Method," Complexity, vol. 2022, no. 1, Jan. 2022, https://doi.org/10.1155/2022/1902166.

[18] M. Liu, I. Dassios, G. Tzounas, and F. Milano, "Model-Independent Derivative Control Delay Compensation Methods for Power Systems," Energies, vol. 13, no. 2, p. 342, Jan. 2020, https://doi.org/10.3390/en13020342.

[19] Y. Koyasako, T. Suzuki, S.-Y. Kim, J.-I. Kani, and J. Terada, "Motion Control System With Time-Varying Delay Compensation for Access Edge Computing," IEEE Access, vol. 9, pp. 90669–90676, Jan. 2021, https://doi.org/10.1109/access.2021.3091707.

[20] D. A. Barkas, G. C. Ioannidis, C. S. Psomopoulos, S. D. Kaminaris, and G. A. Vokas, "Brushed DC Motor Drives for Industrial and Automobile Applications with Emphasis on Control Techniques: A Comprehensive Review," Electronics, vol. 9, no. 6, p. 887, May 2020, https://doi.org/10.3390/electronics9060887.

[21] R. Silva-Ortigoza et al., "Robust Flatness-Based Tracking Control for a 'Full-Bridge Buck Inverter–DC Motor' System," Mathematics, vol. 10, no. 21, p. 4110, Nov. 2022, https://doi.org/10.3390/math10214110.

[22] S. Azizi, M. H. Asemani, N. Vafamand, S. Mobayen, and M. H. Khooban, "A Linear Parameter Varying Control Approach for DC/DC Converters in All-Electric Boats," Complexity, vol. 2021, no. 1, Jan. 2021, https://doi.org/10.1155/2021/8848904.

[23] R. J. Khlif, A. Abid, P. Melchior, and N. Derbel, "UM Shaper Command Inputs for CRONE Control: Application on a DC Motor Bench," Mathematical Problems in Engineering, vol. 2021, pp. 1–11, May 2021, https://doi.org/10.1155/2021/9935875.

[24] Z. Qi, Q. Shi, and H. Zhang, "Tuning of Digital PID Controllers Using Particle Swarm Optimization Algorithm for a CAN-Based DC Motor Subject to Stochastic Delays," IEEE Transactions on Industrial Electronics, vol. 67, no. 7, pp. 5637–5646, Aug. 2019, https://doi.org/10.1109/tie.2019.2934030.

[25] B. Zhu, H. R. Karimi, L. Zhang, and X. Zhao, "Neural network-based adaptive reinforcement learning for optimized backstepping tracking control of nonlinear systems with input delay," Applied Intelligence, vol. 55, no. 2, Dec. 2024, https://doi.org/10.1007/s10489-024-05932-x.

[26] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory. SIAM, 1994, https://doi.org/10.1137/1.9781611970777.

[27] K. K. Afshar, A. Javadi, and M. R. Jahed-Motlagh, "Robust $H\infty$ control of an active suspension system with actuator time delay by predictor feedback," IET Control Theory and Applications, vol. 12, no. 7, pp. 1012–1023, Feb. 2018,, https://doi.org/10.1049/iet-cta.2017.0970.

[28] B. D. O. Anderson and J. B. Moore, Optimal Control: Linear Quadratic Methods. Courier Corporation, 2007.

[29] H. Karami, N. P. Nguyen, H. Ghadiri, S. Mobayen, F. Bayat, P. Skruch, and F. Mostafavi, "LMI-Based Luenberger Observer Design for Uncertain Nonlinear Systems with External Disturbances and Time-Delays," IEEE Access, vol. 11, pp. [pages], July 2023, https://doi.org/10.1109/ACCESS.2023.3293493.

[30] A. Sassi, I. Boussaada, and S.-I. Niculescu, "Observer Design in LTI Time-Delay Systems using Partial Pole Placement with Applications," IFAC PapersOnLine, vol. 55, no. 36, pp. 157–162, 2022, https://doi.org/10.1016/j.ifacol.2022.11.350.